ISC CLASS XII MATHEMATICS (TEST PAPER 12) - SET 12

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. Candidates are required to attempt all questions from **Section A** and **EITHER Section B OR Section C**.
2. All working, including rough work, must be clearly shown. Omission of essential working will result in loss of marks.
3. The maximum mark for any single question is 6.
4. The intended marks for questions or parts of questions are given in brackets [ ].

SECTION A (Compulsory - 65 Marks)

All questions in this section are compulsory. (R&F: 10, Algebra: 10, Calculus: 32, Probability: 13)

Question 1 (10 × 1 Mark = 10 Marks)

Answer the following questions.

1. Let $*$ be an operation on $\mathbb{R}$ defined by $a * b = a + b + 5$. Find the inverse of the element 10. [1]
2. Evaluate: $\sec^2(\tan^{-1}(2))$. [1]
3. State the domain of the function $f(x) = \tan^{-1}\left(\frac{1}{x}\right)$. [1]
4. Let R be a relation on $\mathbb{N}$ defined by xRy if x divides y. Is R a symmetric relation? Justify. [1]
5. Find $\frac{dy}{dx}$ if $y = \sqrt{\sin x}$. [1]
6. If $y = e^{3x} + 2e^{-3x}$, find $\frac{d^2y}{dx^2}$. [1]
7. Write the integrating factor (I.F.) of the differential equation $\frac{dy}{dx} + \frac{1}{x}y = \sin x$. [1]
8. Find the value of $\int_0^2 (x^2 + 1)dx$. [1]
9. If $P(A') = 0.7$, $P(B') = 0.6$, and $P(A \cup B) = 0.6$. Find $P(A \cap B)$. [1]
10. A random variable X has values $0, 1, 2$ with probabilities $0.1, 0.5, 0.4$ respectively. Find the mean $E(X)$. [1]

Question 2 (3 × 2 Marks = 6 Marks)

Answer the following questions.

1. Use differentiation to approximate the change in the area of a circle if its radius changes from 10 cm to 10.1 cm. [2]
2. Find the equation of the normal to the curve $y^2 = 4x$ at the point $(1, 2)$. [2]
3. From a pack of 52 cards, 2 cards are drawn at random without replacement. Find the probability that both are aces. [2]

Question 3 (4 × 4 Marks = 16 Marks)

Answer the following questions.

1. Find the particular solution of the differential equation: $\frac{dy}{dx} = \frac{x+y}{x-y}$, given $y = 0$ when $x = 1$. [4]
2. Find the equation of the tangent and normal to the curve $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at the point $(1, 3)$. [4]
3. Evaluate: $\int \frac{x^2+1}{x^4+x^2+1} dx$. [4]
4. Check if the matrix $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ is involutory. [4]

Question 4 (3 × 6 Marks = 18 Marks)

Answer the following questions.

1. Prove that $\begin{vmatrix} 1 & \alpha & \alpha^2 - \beta\gamma \\ 1 & \beta & \beta^2 - \gamma\alpha \\ 1 & \gamma & \gamma^2 - \alpha\beta \end{vmatrix} = 0$. [6]
2. Show that a right circular cylinder of a given surface area and maximum volume is such that its height is equal to the diameter of the base. [6]
3. Evaluate: $\int_0^{\pi} \frac{x}{a^2 \cos^2 x + b^2 \sin^2 x} dx$. [6]

Question 5 (15 Marks)

Answer the following questions.

1. **(a)** Show that the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 4x - 5$ is invertible. Find its inverse function f^{-1}. [6] **(b)** A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability of getting at least one failure. [6] **(c)** Let A and B be two independent events. If $P(A) = 0.3$ and $P(B) = 0.4$, find the probability of occurrence of at least one of A and B. [3]

SECTION B (Optional - 15 Marks)

Answer all questions from this section. (Unit V: Vectors - 5 Marks; Unit VI: 3D Geometry - 6 Marks; Unit VII: Applications of Integrals - 4 Marks)

Question 6 (5 Marks)

Answer the following questions.

1. If $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$, find the magnitude of the vector $\vec{a} \times \vec{b}$. [2]
2. The position vectors of the vertices of a triangle are $\vec{a}, \vec{b}, \vec{c}$. Find the area of the triangle using the formula $\frac{1}{2}|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$. [3]

Question 7 (10 Marks)

Answer the following questions.

1. Find the equation of the plane containing the lines $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{4}$ and $\frac{x}{1} = \frac{y-2}{-2} = \frac{z-1}{3}$. [6]
2. Using integration, find the area bounded by the curve $y = |x - 1|$, the x-axis, and the lines $x = 0$ and $x = 2$. [4]

SECTION C (Optional - 15 Marks)

Answer all questions from this section. (Unit VIII: Application of Calculus - 5 Marks; Unit IX: Linear Regression - 6 Marks; Unit X: Linear Programming - 4 Marks)

Question 8 (5 Marks)

Answer the following question.

1. The total cost function is $C(x) = 2x^3 - 15x^2 + 36x + 8$. Find the number of units x for which the total cost is minimum. [5]

Question 9 (10 Marks)

Answer the following questions.

1. Solve the following Linear Programming Problem graphically: Minimize $Z = 3x + 5y$ Subject to the constraints:

$$x + 3y \geq 3$$
$$x + y \geq 2$$
$$x, y \geq 0$$

   [4]

2. The regression equations are $3x + 2y = 26$ and $6x + y = 31$. Find the mean of x and y and the coefficient of correlation r. Assume the first equation is the regression line of y on x. [6]