ISC CLASS XII MATHEMATICS (TEST PAPER 9) - SET 09

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. Candidates are required to attempt all questions from **Section A** and **EITHER Section B OR Section C**.
2. All working, including rough work, must be clearly shown. Omission of essential working will result in loss of marks.
3. The maximum mark for any single question is 6.
4. The intended marks for questions or parts of questions are given in brackets [ ].

SECTION A (Compulsory - 65 Marks)

All questions in this section are compulsory. (R&F: 10, Algebra: 10, Calculus: 32, Probability: 13)

Question 1 (10 × 1 Mark = 10 Marks)

Answer the following questions.

1. Let $*$ be a binary operation on $\mathbb{N}$ defined by $a * b = \text{HCF}(a, b)$. Is the operation closed? Justify. [1]
2. Evaluate: $\cos^{-1} x + \sin^{-1} x - \tan^{-1}(1)$. [1]
3. State the domain of the function $f(x) = \sec^{-1}(x - 1)$. [1]
4. Let R be a relation on $\mathbb{Z}$ defined by aRb if $a - b$ is an even integer. Is R a transitive relation? [1]
5. Find $\frac{dy}{dx}$ if $y = x^{\log x}$. [1]
6. Write the general solution of the differential equation $\frac{dy}{dx} = \cos x$. [1]
7. Find the value of $\int_{-1}^{1} \frac{|x|}{x} dx$. [1]
8. Determine if $f(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ 2x - 1 & \text{if } x > 1 \end{cases}$ is continuous at $x = 1$. [1]
9. If $P(A) = 0.6$ and $P(B|A) = 0.5$, find $P(A \cap B)$. [1]
10. If $X \sim B(n, p)$ and $n = 10$, $p = 0.4$, find the mean of the distribution. [1]

Question 2 (3 × 2 Marks = 6 Marks)

Answer the following questions.

1. If $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, find $\frac{d^2y}{dx^2}$. [2]
2. The surface area of a spherical soap bubble is increasing at the rate of 4 cm^2/sec. Find the rate at which its volume is increasing when the radius is 8 cm. [2]
3. Two cards are drawn successively with replacement from a well-shuffled pack of 52 cards. Find the probability that the first card is a king and the second card is a queen. [2]

Question 3 (4×4 Marks = 16 Marks)

Answer the following questions.

1. Find the points on the curve $y = x^3 - 11x + 5$ at which the tangent is parallel to the line $y = x$. [4]
2. Find the particular solution of the differential equation $\frac{dy}{dx} = 1 + x + y + xy$, given $y = 0$ when $x = 1$. [4]
3. Evaluate: $\int \frac{5x+3}{(x-1)(x^2+4)} dx$. [4]
4. If $\begin{vmatrix} 2x+5 & 3 \\ 5x+2 & 9 \end{vmatrix} = 0$, find the value of x. [4]

Question 4 (3×6 Marks = 18 Marks)

Answer the following questions.

1. Show that the semi-vertical angle of a cone of maximum volume and given total surface area is $\sin^{-1}\left(\frac{1}{3}\right)$. [6]
2. Evaluate: $\int \sqrt{9-x^2} \sin^{-1}\left(\frac{x}{3}\right) dx$. [6]
3. Prove that: $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix} = xyz\left(1 + \frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)$. [6]

Question 5 (15 Marks)

Answer the following questions.

1. **(a)** Show that the function $f : \mathbb{R} - \left\{-\frac{4}{3}\right\} \to \mathbb{R} - \left\{\frac{2}{3}\right\}$ defined by $f(x) = \frac{2x-3}{3x+4}$ is both one-one and onto. Find f^{-1}. [6] **(b)** A factory has two machines, Machine A and Machine B. Machine A produces 60% of the items and Machine B produces 40%. Further, 2% of the items produced by Machine A and 1% produced by Machine B are defective. An item is drawn at random and found to be defective. Find the probability that it was produced by Machine B. [6] **(c)** Let A and B be two events such that $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{4}$. Find $P(\text{neither } A \text{ nor } B)$. [3]

SECTION B (Optional - 15 Marks)

Answer all questions from this section. (Unit V: Vectors - 5 Marks; Unit VI: 3D Geometry - 6 Marks; Unit VII: Applications of Integrals - 4 Marks)

Question 6 (5 Marks)

Answer the following questions.

1. Find the area of the parallelogram whose adjacent sides are $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$. [2]
2. Show that the vectors $\vec{a} = \hat{i} + 3\hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} - \hat{k}$, and $\vec{c} = 7\hat{j} + 3\hat{k}$ are coplanar. [3]

Question 7 (10 Marks)

Answer the following questions.

1. Find the distance between the two parallel planes $2x - 2y + z + 3 = 0$ and $4x - 4y + 2z + 5 = 0$. [6]
2. Using integration, find the area of the region bounded by $x = y^2$ and the line $x + y = 2$. [4]

SECTION C (Optional - 15 Marks)

Answer all questions from this section. (Unit VIII: Application of Calculus - 5 Marks; Unit IX: Linear Regression - 6 Marks; Unit X: Linear Programming - 4 Marks)

Question 8 (5 Marks)

Answer the following question.

1. The total cost function is given by $C(x) = 2x^3 - 15x^2 + 30x + 100$. Find the level of output x at which the marginal cost is minimum, and find the minimum marginal cost. [5]

Question 9 (10 Marks)

Answer the following questions.

1. Solve the following Linear Programming Problem graphically: Minimize $Z = 4x + 3y$ Subject to the constraints:
$$\begin{aligned} x + 2y &\geq 10 \\ 3x + 4y &\leq 24 \\ x, y &\geq 0 \end{aligned}$$
[4]

2. The two lines of regression are $2x + 3y = 4$ and $4x + 5y = 12$. Identify which is the regression line of y on x and x on y. Hence, find the coefficient of correlation r. [6]