ISC CLASS XII MATHEMATICS (TEST PAPER 17) - SET 17

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. Candidates are required to attempt all questions from **Section A** and **EITHER Section B OR Section C**.
2. All working, including rough work, must be clearly shown. Omission of essential working will result in loss of marks.
3. The maximum mark for any single question is 6.
4. The intended marks for questions or parts of questions are given in brackets [ ].

SECTION A (Compulsory - 65 Marks)

All questions in this section are compulsory. (R&F: 10, Algebra: 10, Calculus: 32, Probability: 13)

Question 1 (10 × 1 Mark = 10 Marks)

Answer the following questions.

1. Let $*$ be a binary operation on $\mathbb{Q}$ defined by $a * b = a + b + ab$. Find the inverse of the element -2. [1]
2. Find the value of $\sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right)$. [1]
3. State the domain of the function $f(x) = \frac{1}{\sqrt{|x|-x}}$. [1]
4. If $f(x) = \begin{vmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{vmatrix}$, find $f'(x)$. [1]
5. Find $\frac{dy}{dx}$ if $x^y = y^x$. [1]
6. Write the value of $\int_{-\pi}^{\pi} \tan x dx$. [1]
7. What is the order and degree of the differential equation $\frac{dy}{dx} = \sqrt{\frac{y}{x}}$? [1]
8. Find the slope of the tangent to the curve $y = x \log x$ at $x = e$. [1]
9. If $P(A) = 0.5$ and $P(B|A') = 0.2$, and A and B are independent, find $P(B)$. [1]
10. If $X \sim B(n, p)$ has variance 9 and $p = 0.4$, find the value of n. [1]

Question 2 (3 × 2 Marks = 6 Marks)

Answer the following questions.

1. If $x = a(\cos t + t\sin t)$ and $y = a(\sin t - t\cos t)$, find $\frac{d^2y}{dx^2}$. [2]

2. The edges of a variable cube are increasing at the rate of 3 cm/s. How fast is the volume of the cube increasing when the edge is 10 cm? [2]

3. A fair coin is tossed until a head appears or 4 tosses are completed. Find the probability distribution of the number of tosses. [2]

Question 3 (4×4 Marks $= 16$ Marks)

Answer the following questions.

1. Show that the function $f(x) = 2x^3 - 3x^2 - 12x + 6$ has local maxima at $x = -1$ and local minima at $x = 2$. [4]

2. Find the particular solution of the differential equation: $\frac{dy}{dx} = 1 + x + y^2 + xy^2$, given $y(0) = 1$. [4]

3. Evaluate: $\int \frac{dx}{x(x^3+1)}$. [4]

4. Find the inverse of the matrix $A = \begin{pmatrix} 3 & 1 & 2 \\ 2 & 1 & 0 \\ 1 & 2 & 3 \end{pmatrix}$ using the adjoint method. [4]

Question 4 (3×6 Marks $= 18$ Marks)

Answer the following questions.

1. A wire of length L is cut into two pieces. One piece is bent into a circle and the other into a square. Where should the wire be cut so that the sum of the areas enclosed by both is minimum? [6]

2. Evaluate: $\int_0^1 \frac{\log(1+x)}{1+x^2} dx$. [6]

3. Solve the system of linear equations using the matrix inverse method: [6]

$$\begin{aligned} x + y + z &= 6 \\ y - z &= 2 \\ 2x - 3y + 4z &= 9 \end{aligned}$$

Question 5 (15 Marks)

Answer the following questions.

1. **(a)** Prove that: $2\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \frac{\pi}{4}$. [6] **(b)** An electronic manufacturer has two manufacturing plants, Plant A and Plant B. Plant A produces 60% of the output and Plant B produces 40%. 2% of the items produced by A and 3% of the items produced by B are defective. An item is selected at random and found to be non-defective. Find the probability that it was produced by Plant B. [6] **(c)** Show that the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \frac{x}{1+|x|}$ is one-one. [3]

SECTION B (Optional - 15 Marks)

Answer all questions from this section. (Unit V: Vectors - 5 Marks; Unit VI: 3D Geometry - 6 Marks; Unit VII: Applications of Integrals - 4 Marks)

Question 6 (5 Marks)

Answer the following questions.

1. Find the area of the parallelogram whose adjacent sides are the vectors $\vec{a} = 3\hat{i} + \hat{j} + 4\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$. [2]
2. If the magnitude of the scalar projection of the vector $\lambda\hat{i} + \hat{j} + 4\hat{k}$ on the vector $2\hat{i} + 6\hat{j} + 3\hat{k}$ is 1, find the value of λ. [3]

Question 7 (10 Marks)

Answer the following questions.

1. Find the equation of the plane passing through the points $(3, 4, 1)$ and $(0, 1, 0)$ and parallel to the line $\frac{x+3}{3} = \frac{y-3}{2} = \frac{z-2}{5}$. [6]
2. Using integration, find the area bounded by the curve $x = y^2$ and the line $x = 4$. [4]

SECTION C (Optional - 15 Marks)

Answer all questions from this section. (Unit VIII: Application of Calculus - 5 Marks; Unit IX: Linear Regression - 6 Marks; Unit X: Linear Programming - 4 Marks)

Question 8 (5 Marks)

Answer the following question.

1. The total cost function for a commodity is given by $C(x) = \frac{1}{3}x^3 - 5x^2 + 28x + 10$. Find the level of output x at which the marginal cost is minimum, and find the minimum marginal cost. [5]

Question 9 (10 Marks)

Answer the following questions.

1. Solve the following Linear Programming Problem graphically: Maximize $Z = 3x + 4y$ Subject to the constraints:
$$\begin{aligned} x + y &\leq 4 \\ x &\geq 0 \\ y &\geq 0 \\ x, y &\geq 0 \end{aligned}$$
[4]
2. The two lines of regression are $4x + 3y + 7 = 0$ and $3x + 4y + 8 = 0$. Find the coefficient of correlation r. If $\sigma_x = 2$, find σ_y. (Assume $4x + 3y + 7 = 0$ is the regression line of y on x). [6]