ISC CLASS XII MATHEMATICS (TEST PAPER 15) - SET 15

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. Candidates are required to attempt all questions from **Section A** and **EITHER Section B OR Section C**.
2. All working, including rough work, must be clearly shown. Omission of essential working will result in loss of marks.
3. The maximum mark for any single question is 6.
4. The intended marks for questions or parts of questions are given in brackets [ ].

SECTION A (Compulsory - 65 Marks)

All questions in this section are compulsory. (R&F: 10, Algebra: 10, Calculus: 32, Probability: 13)

Question 1 (10 $\times$ 1 Mark = 10 Marks)

Answer the following questions.

1. Let $*$ be an operation on $\mathbb{Z}$ defined by $a * b = a + b - ab$. Check if 0 is the identity element for $*$. [1]
2. Find the principal value of $\tan^{-1}\left(\tan \frac{3\pi}{4}\right)$. [1]
3. State the range of the function $f(x) = \cos^{-1}(1 - 2x^2)$. [1]
4. Let R be a relation on A. If for all $a, b \in A$, $(a, b) \in R \iff (b, a) \in R$, what property does R possess? [1]
5. Find $\frac{dy}{dx}$ if $y = \sqrt{\tan x}$. [1]
6. Write the order and degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^2 - 4\frac{d^2y}{dx^2} + 5y = 0$. [1]
7. Determine the value of k for which $f(x) = \begin{cases} \frac{x^2-1}{x-1} & \text{if } x \neq 1 \\ k & \text{if } x = 1 \end{cases}$ is continuous at $x = 1$. [1]
8. Evaluate: $\int_{-2}^{2} (x^3 + \cos x) dx$. [1]
9. If $P(A) = 0.4$ and $P(B) = 0.5$, and A and B are independent, find $P(A' \cup B')$. [1]
10. If a random variable $X \sim B(n, p)$ has mean 12 and variance 4, find the value of n. [1]

Question 2 (3 $\times$ 2 Marks = 6 Marks)

Answer the following questions.

1. If $y = x^4 + 5x^2 - 7$, find $\frac{d^2y}{dx^2}$ at $x = 2$. [2]

2. The radius of a metal plate is expanding at the rate of 0.01 cm/s. Find the rate at which the area is expanding when the radius is 10 cm. [2]

3. A bag contains 5 red and 5 black balls. Two balls are drawn at random without replacement. Find the probability that they are of different colours. [2]

Question 3 (4 × 4 Marks = 16 Marks)

Answer the following questions.

1. Verify Rolle's Theorem for the function $f(x) = x^2 - 4x + 3$ on the interval $[1, 3]$. [4]

2. Find the particular solution of the differential equation: $\frac{dy}{dx} + y \cot x = 2x + x^2 \cot x$, given $y(\frac{\pi}{2}) = 0$. [4]

3. Evaluate: $\int \frac{dx}{\sqrt{x^2-4x+8}}$. [4]

4. Given $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix}$. Find a matrix X such that $XA = B$. [4]

Question 4 (3 × 6 Marks = 18 Marks)

Answer the following questions.

1. Prove that: $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$. [6]

2. Evaluate: $\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx$. [6]

3. Find the dimensions of the rectangle of perimeter 36 cm which will sweep out a maximum volume when revolved about one of its sides. [6]

Question 5 (15 Marks)

Answer the following questions.

1. **(a)** Let L be the set of all lines in a plane and R be a relation on L defined as $R = \{(l_1, l_2) : l_1 \text{ is parallel to } l_2\}$. Show that R is an equivalence relation. [6] **(b)** The probability that a student gets Grade A in Mathematics is 0.6 and the probability that he gets Grade A in Physics is 0.5. The probability that he gets Grade A in both subjects is 0.3. Find the probability that he gets Grade A in at least one of the subjects. [4] **(c)** A die is thrown 6 times. If getting an odd number is a success, find the probability of getting at most 2 successes. [5]

SECTION B (Optional - 15 Marks)

Answer all questions from this section. (Unit V: Vectors - 5 Marks; Unit VI: 3D Geometry - 6 Marks; Unit VII: Applications of Integrals - 4 Marks)

Question 6 (5 Marks)

Answer the following questions.

1. If $\vec{a} = 2\hat{i} + \hat{j} + k\hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$ are perpendicular to each other, find the value of k. [2]
2. Find the area of the triangle whose vertices are $A(1, 1, 1)$, $B(1, 2, 3)$, and $C(2, 3, 1)$ using vector methods. [3]

Question 7 (10 Marks)

Answer the following questions.

1. Find the equation of the plane passing through the point $(1, 0, -2)$ and perpendicular to each of the planes $2x + y - z = 2$ and $x - y - z = 3$. [6]
2. Using integration, find the area bounded by the curve $y = |x + 1|$, the x-axis, and the lines $x = -3$ and $x = 1$. [4]

SECTION C (Optional - 15 Marks)

Answer all questions from this section. (Unit VIII: Application of Calculus - 5 Marks; Unit IX: Linear Regression - 6 Marks; Unit X: Linear Programming - 4 Marks)

Question 8 (5 Marks)

Answer the following question.

1. The total cost function is $C(x) = 2x^3 - 15x^2 + 36x + 50$. Find the value of x for which the marginal cost is minimum, and find the minimum marginal cost. [5]

Question 9 (10 Marks)

Answer the following questions.

1. Solve the following Linear Programming Problem graphically: Minimize $Z = 2x + y$ Subject to the constraints:
$$x + 2y \leq 8$$
$$x + y \leq 5$$
$$x, y \geq 0$$
[4]

2. The regression coefficient of y on x is $b_{yx} = 0.8$, and the correlation coefficient $r = 0.6$. If the standard deviation of y is $\sigma_y = 5$, find the standard deviation of x, σ_x. [6]