ISC CLASS XII MATHEMATICS (TEST PAPER 5) - SET 05

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. Candidates are required to attempt all questions from **Section A** and **EITHER Section B OR Section C**.
2. All working, including rough work, must be clearly shown. Omission of essential working will result in loss of marks.
3. The maximum mark for any single question is 6.
4. The intended marks for questions or parts of questions are given in brackets [ ].

SECTION A (Compulsory - 65 Marks)

All questions in this section are compulsory. (R&F: 10, Algebra: 10, Calculus: 32, Probability: 13)

Question 1 (10 $\times$ 1 Mark = 10 Marks)

Answer the following questions.

1. Let $*$ be an operation on $\mathbb{R}$ defined by $a * b = a + b + 1$. Check if $*$ is associative. [1]
2. Find the value of $\tan\left(\frac{1}{2}\cos^{-1}\frac{3}{5}\right)$. [1]
3. Give an example of a function $f : \{1, 2, 3\} \to \{1, 2, 3\}$ which is not one-one. [1]
4. Find $g \circ f(x)$ if $f(x) = \frac{x}{\sqrt{1+x^2}}$ and $g(x) = \frac{x}{\sqrt{1-x^2}}$. [1]
5. Find $\frac{dy}{dx}$ if $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$. [1]
6. Find the value of $\int_{-1}^{1}(x^3 + x^5)dx$. [1]
7. Find the value of a for which the function $f(x) = \begin{cases} ax + 5 & \text{if } x \leq 2 \\ x - 1 & \text{if } x > 2 \end{cases}$ is continuous at $x = 2$. [1]
8. If $P(A) = 0.4$, $P(B) = 0.5$, and $P(A \cap B) = 0.3$. Find $P(A|B)$. [1]
9. If a random variable X follows a Binomial distribution $B(6, p)$ and its variance is $\frac{12}{9}$, find the value of p. [1]
10. If $A = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$, find $M_{12} + C_{21}$, where M_{ij} is the minor and C_{ij} is the cofactor. [1]

Question 2 (3 $\times$ 2 Marks = 6 Marks)

Answer the following questions.

1. If $y = x^{\cos x}$, find $\frac{dy}{dx}$. [2]

2. Prove that $f(x) = \frac{1}{x^2+1}$ is a strictly decreasing function for $x > 0$. [2]
3. Three unbiased coins are tossed simultaneously. Find the probability that the first coin shows head, given that there is at least one tail. [2]

Question 3 (4 × 4 Marks = 16 Marks)

Answer the following questions.

1. If $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$, find $\text{adj}(A)$ and check if $A \cdot \text{adj}(A) = |A|I$. [4]
2. Evaluate: $\int \frac{x \cos x}{\sin^2 x} dx$. [4]
3. Show that Rolle's theorem is not applicable to $f(x) = |x - 1|$ in $[0, 2]$. [4]
4. The radius of a sphere is measured to be 7 m with an error of 0.02 m. Find the approximate error in calculating its surface area. [4]

Question 4 (3 × 6 Marks = 18 Marks)

Answer the following questions.

1. Prove that: $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$. [6]
2. A straight line L passes through $(1, 2)$ and is such that the segment of L between the axes is minimum. Find the equation of the line L. [6]
3. Evaluate: $\int_0^{\pi/2} \frac{1}{4+5\cos x} dx$. [6]

Question 5 (15 Marks)

Answer the following questions.

1. **(a)** Prove that $2\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \frac{\pi}{4}$. [6] **(b)** Find the mean and variance of the number of heads in three tosses of a fair coin. [6] **(c)** Solve the differential equation: $x\frac{dy}{dx} - y = x^2 e^x$. [3]

SECTION B (Optional - 15 Marks)

Answer all questions from this section. (Unit V: Vectors - 5 Marks; Unit VI: 3D Geometry - 6 Marks; Unit VII: Applications of Integrals - 4 Marks)

Question 6 (5 Marks)

Answer the following questions.

1. Find the distance of the point $(3, -2, 1)$ from the plane $2x - y + 2z + 3 = 0$. [2]
2. Find the value of λ such that the four points $A(4, 5, 1)$, $B(0, -1, -1)$, $C(3, 9, 4)$, and $D(-4, 4, 4)$ are coplanar. [3]

Question 7 (10 Marks)

Answer the following questions.

1. Find the cartesian equation of the line passing through the points $A(1, 2, 3)$ and $B(-1, 0, 4)$. Find the angle between this line and the line $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{-2}$. [6]
2. Using integration, find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$. [4]

SECTION C (Optional - 15 Marks)

Answer all questions from this section. (Unit VIII: Application of Calculus - 5 Marks; Unit IX: Linear Regression - 6 Marks; Unit X: Linear Programming - 4 Marks)

Question 8 (5 Marks)

Answer the following question.

1. A company is preparing a budget for the next year. It estimates its fixed costs to be Rs. 18,000 and variable costs at Rs. 1.60 per unit of output. If the selling price is Rs. 4 per unit, find the total cost function $C(x)$, the total revenue function $R(x)$, and the break-even point. [5]

Question 9 (10 Marks)

Answer the following questions.

1. Solve the following Linear Programming Problem graphically: Maximize $Z = 3x + 4y$ Subject to the constraints:

$$x + y \leq 4$$
$$x \geq 0$$
$$y \geq 0$$

   [4]

2. From the data given below, find the two regression equations and the estimated value of y when $x = 18$:

x	10	15	20	25	30
y	30	35	45	50	60

   [6]