ISC CLASS XII MATHEMATICS (TEST PAPER 11) - SET 11

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. Candidates are required to attempt all questions from **Section A** and **EITHER Section B OR Section C**.
2. All working, including rough work, must be clearly shown. Omission of essential working will result in loss of marks.
3. The maximum mark for any single question is 6.
4. The intended marks for questions or parts of questions are given in brackets [ ].

SECTION A (Compulsory - 65 Marks)

All questions in this section are compulsory. (R&F: 10, Algebra: 10, Calculus: 32, Probability: 13)

Question 1 (10 × 1 Mark = 10 Marks)

Answer the following questions.

1. Let $*$ be a binary operation on $\mathbb{Z}$ defined by $a * b = a + 3b$. Is the operation closed on $\mathbb{Z}$? Justify. [1]
2. Evaluate: $\cot(\tan^{-1}(\frac{1}{2}))$. [1]
3. State the range of the function $f(x) = \sin^{-1}(2x^2 - 1)$. [1]
4. Define the equivalence class of an element a with respect to an equivalence relation R on set A. [1]
5. Find $\frac{dy}{dx}$ if $\log(xy) = x^2$. [1]
6. Write the type of solution obtained when we use the Variable Separable Method to solve a differential equation. [1]
7. Determine if $f(x) = \begin{cases} \frac{\log(1+2x)}{x} & \text{if } x \neq 0 \\ 2 & \text{if } x = 0 \end{cases}$ is continuous at $x = 0$. [1]
8. Find the area bounded by the curve $y = 2x$ from $x = 0$ to $x = 2$ (Integral expression only). [1]
9. If A and B are independent events, simplify $P(A|B) + P(A'|B)$. [1]
10. If X is a random variable, write the formula for variance in terms of $E(X)$ and $E(X^2)$. [1]

Question 2 (3 × 2 Marks = 6 Marks)

Answer the following questions.

1. If $y = x^5 + 3x^3 + 2$, find $\frac{d^2y}{dx^2}$ at $x = 1$. [2]

2. A particle moves along the curve $6y = x^3 + 2$. Find the point on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate. [2]
3. A bag contains 5 white and 7 black balls. Two balls are drawn without replacement. Find the probability that both are of the same color. [2]

Question 3 (4 × 4 Marks = 16 Marks)

Answer the following questions.

1. Find the intervals in which the function $f(x) = 2x^3 - 9x^2 + 12x + 15$ is strictly increasing or strictly decreasing. [4]
2. Find the particular solution of the differential equation: $\frac{dy}{dx} + y \cot x = 2x + x^2 \cot x$, given $y(\frac{\pi}{2}) = 0$. [4]
3. Evaluate: $\int \frac{dx}{\sqrt{9-x^2}}$. [4]
4. Given the matrices $A = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$. Find a matrix X such that $A^T X = B$. [4]

Question 4 (3 × 6 Marks = 18 Marks)

Answer the following questions.

1. Show that the triangle of maximum area that can be inscribed in a circle of radius r is an equilateral triangle. [6]
2. Evaluate: $\int \log(x^2 + a^2) dx$. [6]
3. Solve the system of linear equations using the matrix inverse method: [6]

$$\begin{aligned} x + 2y - 3z &= 6 \\ 3x + 2y - 2z &= 3 \\ 2x - y + z &= 2 \end{aligned}$$

Question 5 (15 Marks)

Answer the following questions.

1. **(a)** Prove that: $\tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3) = \pi$. [6] **(b)** In a test, a student either guesses the answer or knows the answer. The probability that he guesses is $\frac{1}{3}$ and the probability that he knows the answer is $\frac{2}{3}$. Assuming that a student who guesses has $\frac{1}{4}$ chance of being correct, what is the probability that the student knows the answer, given that he answered it correctly? [6] **(c)** Let R be a relation on $\mathbb{Z}$ defined by $(a, b) \in R$ if $a - b$ is divisible by 5. Show that R is an equivalence relation. [3]

SECTION B (Optional - 15 Marks)

Answer all questions from this section. (Unit V: Vectors - 5 Marks; Unit VI: 3D Geometry - 6 Marks; Unit VII: Applications of Integrals - 4 Marks)

Question 6 (5 Marks)

Answer the following questions.

1. If $|\vec{a}| = 2$, $|\vec{b}| = 3$, and $\vec{a} \cdot \vec{b} = 4$, find $|\vec{a} - \vec{b}|$. [2]
2. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, and $|\vec{a}| = 3, |\vec{b}| = 4, |\vec{c}| = 5$, find $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$. [3]

Question 7 (10 Marks)

Answer the following questions.

1. Find the foot of the perpendicular drawn from the point $P(0, 2, 3)$ to the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$. [6]
2. Using integration, find the area of the region bounded by the parabola $y^2 = 4x$ and the line $x = 3$. [4]

SECTION C (Optional - 15 Marks)

Answer all questions from this section. (Unit VIII: Application of Calculus - 5 Marks; Unit IX: Linear Regression - 6 Marks; Unit X: Linear Programming - 4 Marks)

Question 8 (5 Marks)

Answer the following question.

1. The total cost $C(x)$ and the revenue $R(x)$ functions for a firm are $C(x) = 2x^2 + 15x + 300$ and $R(x) = 80x - 2x^2$. Find the level of output x at which the profit is maximum. Find the maximum profit. [5]

Question 9 (10 Marks)

Answer the following questions.

1. Solve the following Linear Programming Problem graphically: Minimize $Z = 3x + 2y$ Subject to the constraints:
$$x + y \leq 8$$
$$x + 2y \geq 4$$
$$x, y \geq 0$$
[4]
2. The regression equations are $x = 0.8y + a$ and $y = 0.4x + b$. Find the value of the correlation coefficient r. Given that $\sigma_x = 3$, find the value of σ_y. [6]