ISC CLASS XII MATHEMATICS (TEST PAPER 7) - SET 07

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. Candidates are required to attempt all questions from **Section A** and **EITHER Section B OR Section C**.
2. All working, including rough work, must be clearly shown. Omission of essential working will result in loss of marks.
3. The maximum mark for any single question is 6.
4. The intended marks for questions or parts of questions are given in brackets [ ].

SECTION A (Compulsory - 65 Marks)

All questions in this section are compulsory. (R&F: 10, Algebra: 10, Calculus: 32, Probability: 13)

Question 1 (10 × 1 Mark = 10 Marks)

Answer the following questions.

1. Let $*$ be a binary operation on $\mathbb{Z}$ defined by $a * b = a + b - 4$. Find the inverse of the element 5. [1]
2. Evaluate: $\tan^{-1}(2) + \tan^{-1}(3)$. [1]
3. If $f(x) = 8x^3$ and $g(x) = x^{1/3}$, find $f \circ g(x)$. [1]
4. Let R be an equivalence relation on a set A. State the property relating the union of two distinct equivalence classes $[a]$ and $[b]$. [1]
5. Find $\frac{dy}{dx}$ if $2x + 3y = \sin y$. [1]
6. Evaluate: $\int_0^{\pi/2} \sin^3 x dx$. [1]
7. Find the order of the differential equation of the family of curves $y = Ae^x + Be^{2x}$. [1]
8. Find $\frac{dy}{dx}$ if $y = 5^{x^2}$. [1]
9. If $P(A \cup B) = 0.8$, $P(A) = 0.3$, $P(B) = 0.5$, find $P(B|A)$. [1]
10. If $A = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$, find $\det(A^{-1})$. [1]

Question 2 (3 × 2 Marks = 6 Marks)

Answer the following questions.

1. If $x = a(\cos t + t)$ and $y = a(\sin t - 1)$, find $\frac{d^2y}{dx^2}$. [2]
2. Find the intervals in which the function $f(x) = \sin x + \cos x$ is strictly increasing in $[0, 2\pi]$. [2]
3. A point is chosen at random from the square with vertices $(0, 0), (1, 0), (1, 1), (0, 1)$. Find the probability that the chosen point lies inside the circle $x^2 + y^2 = 1$. [2]

Question 3 (4×4 Marks $= 16$ Marks)

Answer the following questions.

1. Given $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$. Find the minor M_{13}, the cofactor C_{21}, and verify that $a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13} = |A|$. [4]
2. Solve the differential equation: $x\frac{dy}{dx} = y(\log y - \log x + 1)$. [4]
3. Evaluate: $\int \frac{x^2+1}{(x^2+2)(x^2+3)}dx$. [4]
4. The radius of a right circular cylinder is 3 cm and its height is 5 cm. Find the approximate change in its volume when the radius is increased by 0.03 cm and the height is decreased by 0.01 cm. [4]

Question 4 (3×6 Marks $= 18$ Marks)

Answer the following questions.

1. Find the coordinates of the point on the parabola $y = x^2 + 7x + 2$ which is nearest to the line $y = 3x - 3$. [6]
2. Evaluate: $\int e^{2x}\sin(x/2)dx$. [6]
3. Prove that $A \cdot \text{adj}(A) = |\text{A}|I$, where $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 5 & 0 \\ 2 & 4 & 3 \end{pmatrix}$. [6]

Question 5 (15 Marks)

Answer the following questions.

1. **(a)** Prove that $\cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \cos^{-1}\left(\frac{33}{65}\right)$. [6] **(b)** A person has undertaken a construction job. The probabilities are 0.65 that there will be a strike, 0.80 that the construction job will be completed on time if there is no strike, and 0.32 that the construction job will be completed on time if there is a strike. Find the probability that the job will be completed on time. [4] **(c)** In a group of students, two are selected. The probability that the first student is a boy is 0.7 and the probability that the second student is a boy, given that the first is a boy, is 0.8. Find the probability that both students selected are boys. [5]

SECTION B (Optional - 15 Marks)

Answer all questions from this section. (Unit V: Vectors - 5 Marks; Unit VI: 3D Geometry - 6 Marks; Unit VII: Applications of Integrals - 4 Marks)

Question 6 (5 Marks)

Answer the following questions.

1. Find the angle between the diagonals of the parallelogram whose adjacent sides are $2\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} - \hat{j} + 2\hat{k}$. [2]
2. Find the area of a triangle having the points $A(1, 1, 1)$, $B(1, 2, 3)$, and $C(2, 3, 1)$ as its vertices using vector methods. [3]

Question 7 (10 Marks)

Answer the following questions.

1. Find the angle between the planes $x - 2y + 2z = 5$ and $2x + 2y + z = 1$. Find the distance of the point $(1, 0, 1)$ from the first plane. [6]
2. Using integration, find the area bounded by the curve $y = x^2 - 4x$ and the x-axis. [4]

SECTION C (Optional - 15 Marks)

Answer all questions from this section. (Unit VIII: Application of Calculus - 5 Marks; Unit IX: Linear Regression - 6 Marks; Unit X: Linear Programming - 4 Marks)

Question 8 (5 Marks)

Answer the following question.

1. The total cost $C(x)$ for producing x units of a product is $C(x) = 200 + 0.05x + 0.0001x^2$. Find the marginal cost (MC), average cost (AC), and the value of x for which the marginal cost is minimum. [5]

Question 9 (10 Marks)

Answer the following questions.

1. Solve the following Linear Programming Problem graphically: Minimize $Z = 5x + 10y$ Subject to the constraints:

$$\begin{aligned} x + 2y &\leq 120 \\ x + y &\geq 60 \\ x - 2y &\geq 0 \\ x, y &\geq 0 \end{aligned}$$

   [4]

2. The regression line of y on x is $y = 3x + 1$ and the regression line of x on y is $x = 0.5y - 0.5$. Find the correlation coefficient r between x and y and the variance of y if the standard deviation of x is $\sigma_x = 2$. [6]