ISC CLASS XII MATHEMATICS (TEST PAPER 16) - SET 16

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. Candidates are required to attempt all questions from **Section A** and **EITHER Section B OR Section C**.
2. All working, including rough work, must be clearly shown. Omission of essential working will result in loss of marks.
3. The maximum mark for any single question is 6.
4. The intended marks for questions or parts of questions are given in brackets [ ].

SECTION A (Compulsory - 65 Marks)

All questions in this section are compulsory. (R&F: 10, Algebra: 10, Calculus: 32, Probability: 13)

Question 1 (10 × 1 Mark = 10 Marks)

Answer the following questions.

1. Let $*$ be an operation on $\mathbb{Q}$ defined by $a * b = \frac{a+b}{2}$. Check if $*$ is commutative. [1]
2. Find the value of $\tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3)$. [1]
3. Let $f : \{1, 2, 3\} \to \{a, b, c\}$ be a bijective function. Write the number of such possible functions. [1]
4. If $f(x) = |x| - x$, find $f \circ f(x)$. [1]
5. Find $\frac{dy}{dx}$ if $x\sqrt{1+y} + y\sqrt{1+x} = 0$. [1]
6. Write the value of $\int_0^{\pi/2} \log(\tan x)dx$. [1]
7. Write the order and degree of the differential equation $\sqrt{\frac{d^2y}{dx^2} + y} = \frac{dy}{dx}$. [1]
8. Find the value of λ for which the function $f(x) = \lambda(x^2 + 1)$ is strictly increasing on $[0, 1]$. [1]
9. If $P(A) = 0.8$ and $P(B|A) = 0.4$, find $P(A \cap B)$. [1]
10. The mean of a Binomial distribution $B(n, p)$ is 4 and $n = 16$. Find the variance. [1]

Question 2 (3 × 2 Marks = 6 Marks)

Answer the following questions.

1. If $y = (\sin x)^{\log x}$, find $\frac{dy}{dx}$. [2]
2. Find the equation of all lines having slope -1 that are tangent to the curve $y = \frac{1}{x-1}$. [2]
3. A family has two children. Find the probability that both are boys, given that at least one of them is a boy. [2]

Question 3 (4×4 Marks = 16 Marks)

Answer the following questions.

1. Show that $\tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}$. [4]
2. Find the matrix A such that $A\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$. [4]
3. Evaluate: $\int \frac{dx}{3+2\sin x+\cos x}$. [4]
4. Find the maximum value of the function $f(x) = \sin x + \cos x$. [4]

Question 4 (3×6 Marks = 18 Marks)

Answer the following questions.

1. Prove that: $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$. [6]
2. A window is in the form of a rectangle surmounted by a semi-circle. If the total perimeter of the window is 30 m, find the dimensions of the window to admit maximum light (maximum area). [6]
3. Evaluate: $\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx$. [6]

Question 5 (15 Marks)

Answer the following questions.

1. **(a)** Prove that the greatest integer function $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = [x]$ is neither one-one nor onto. [6] **(b)** Solve the differential equation: $x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right)$. [6] **(c)** A card is drawn from a well-shuffled pack of 52 cards. Find the probability distribution of the number of aces drawn. [3]

SECTION B (Optional - 15 Marks)

Answer all questions from this section. (Unit V: Vectors - 5 Marks; Unit VI: 3D Geometry - 6 Marks; Unit VII: Applications of Integrals - 4 Marks)

Question 6 (5 Marks)

Answer the following questions.

1. Find the angle between the vectors $\vec{a} = 2\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = 6\hat{i} - 3\hat{j} + 2\hat{k}$. [2]
2. Find the volume of the parallelopiped whose coterminous edges are $3\hat{i} + 2\hat{j} + 5\hat{k}$, $2\hat{i} - \hat{j} + 3\hat{k}$, and $\hat{i} - 2\hat{j} + 4\hat{k}$. [3]

Question 7 (10 Marks)

Answer the following questions.

1. Find the equation of the plane passing through the intersection of the planes $2x + y - z = 3$ and $5x - 3y + 4z + 9 = 0$ and parallel to the line $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z-5}{5}$. [6]
2. Using integration, find the area bounded by the curve $x = 4 - y^2$ and the y-axis. [4]

SECTION C (Optional - 15 Marks)

Answer all questions from this section. (Unit VIII: Application of Calculus - 5 Marks; Unit IX: Linear Regression - 6 Marks; Unit X: Linear Programming - 4 Marks)

Question 8 (5 Marks)

Answer the following question.

1. The cost of producing x units of an item is given by $C(x) = x^3 - 10x^2 + 15x + 500$. Find the level of output x at which the marginal cost is minimum, and find the minimum marginal cost. [5]

Question 9 (10 Marks)

Answer the following questions.

1. Solve the following Linear Programming Problem graphically: Maximize $Z = 4x + y$ Subject to the constraints:
$$\begin{aligned} x + y &\leq 50 \\ x &\geq 10 \\ y &\geq 0 \end{aligned}$$
   [4]

2. Given $b_{yx} = 0.5$ and $b_{xy} = 0.7$. Find the coefficient of correlation r and identify a contradiction in the given data. Assuming the data is slightly off and $b_{xy} = 0.3$, find r. [6]