ISC CLASS XII MATHEMATICS (TEST PAPER 13) - SET 13

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. Candidates are required to attempt all questions from **Section A** and **EITHER Section B OR Section C**.
2. All working, including rough work, must be clearly shown. Omission of essential working will result in loss of marks.
3. The maximum mark for any single question is 6.
4. The intended marks for questions or parts of questions are given in brackets [ ].

SECTION A (Compulsory - 65 Marks)

All questions in this section are compulsory. (R&F: 10, Algebra: 10, Calculus: 32, Probability: 13)

Question 1 (10×1 Mark = 10 Marks)

Answer the following questions.

1. Let $*$ be a binary operation on $\mathbb{Z}^+$ defined by $a * b = \min(a, b)$. Check if $*$ is associative. [1]
2. Evaluate: $\tan^{-1}(2) + \cot^{-1}(2)$. [1]
3. State the domain of $f(x) = \sin^{-1}(1 - x)$. [1]
4. Check if the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 2 - 3x$ is onto. [1]
5. Find $\frac{dy}{dx}$ if $y = e^{\sin x}$. [1]
6. Find the value of $\int_0^1 \frac{x}{x^2+1} dx$. [1]
7. Write the value of $\int_{-1}^1 (x^2 + x^4) dx$. [1]
8. Write the integrating factor (I.F.) of the differential equation $\frac{dy}{dx} - y \tan x = \sin x$. [1]
9. If A and B are independent events with $P(A) = 0.5$ and $P(B) = 0.6$, find $P(A'|B)$. [1]
10. If $X \sim B(n, p)$, $n = 5$ and $p = 0.2$, find $E(X)$. [1]

Question 2 (3×2 Marks = 6 Marks)

Answer the following questions.

1. If $y = \log(\log x)$, find $\frac{d^2y}{dx^2}$. [2]
2. Find the slope of the tangent to the curve $x^2 + 3xy + y^2 = 5$ at the point $(1, 1)$. [2]
3. A bag contains 5 red and 3 black balls. Two balls are drawn at random without replacement. Find the probability that the second ball drawn is red, given the first ball drawn was black. [2]

Question 3 (4×4 Marks = 16 Marks)

Answer the following questions.

1. Find the approximate value of $\sqrt{49.5}$ using differentiation. [4]
2. Find the particular solution of the differential equation: $\cos x \frac{dy}{dx} + y \sin x = \sec x$, given $y(0) = 1$. [4]
3. Evaluate: $\int \frac{x^2+x+1}{(x+1)(x-2)^2} dx$. [4]
4. If $A = \begin{pmatrix} 2 & 1 \\ 3 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 \\ 4 & 2 \end{pmatrix}$. Find the matrix X such that $2X = (AB)^T$. [4]

Question 4 (3×6 Marks = 18 Marks)

Answer the following questions.

1. Find the coordinates of the point on the curve $y^2 = 4x$ which is at the least distance from the origin. [6]
2. Evaluate: $\int \frac{dx}{\sqrt{2x^2+3x-2}}$. [6]
3. Find the inverse of the matrix $A = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{pmatrix}$ using elementary row transformations. [6]

Question 5 (15 Marks)

Answer the following questions.

1. **(a)** Prove that $\sin^{-1}\left(\frac{12}{13}\right) + \cos^{-1}\left(\frac{4}{5}\right) = \sin^{-1}\left(\frac{63}{65}\right)$. [6] **(b)** An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident involving a scooter, car, and truck driver is $0.01, 0.03$ and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a car driver? [6] **(c)** Let R be a relation on the set $A = \{1, 2, 3, 4, 5, 6\}$ defined by $R = \{(x, y) : y = x + 1\}$. Determine if R is reflexive. [3]

SECTION B (Optional - 15 Marks)

Answer all questions from this section. (Unit V: Vectors - 5 Marks; Unit VI: 3D Geometry - 6 Marks; Unit VII: Applications of Integrals - 4 Marks)

Question 6 (5 Marks)

Answer the following questions.

1. If $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$ are orthogonal, find the value of λ. [2]
2. Find the value of λ such that the vectors $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = \lambda\hat{i} + 4\hat{j} + 7\hat{k}$, and $\vec{c} = \hat{i} - 3\hat{j} - 2\hat{k}$ are coplanar. [3]

Question 7 (10 Marks)

Answer the following questions.

1. Find the equation of the plane passing through the point $(3, -3, 1)$ and perpendicular to the planes $2x + y - 2z = 5$ and $3x - 6y - 2z = 7$. [6]
2. Using integration, find the area bounded by the curve $x = y^2$ and the y-axis between the lines $y = 1$ and $y = 4$. [4]

SECTION C (Optional - 15 Marks)

Answer all questions from this section. (Unit VIII: Application of Calculus - 5 Marks; Unit IX: Linear Regression - 6 Marks; Unit X: Linear Programming - 4 Marks)

Question 8 (5 Marks)

Answer the following question.

1. The demand function for a certain product is $p = 150 - 2x$, and the cost function is $C(x) = 2x^2 + 10x + 50$. Find the number of units x that should be produced to maximize the profit. [5]

Question 9 (10 Marks)

Answer the following questions.

1. Solve the following Linear Programming Problem graphically: Maximize $Z = 6x + 5y$ Subject to the constraints:
$$x + y \leq 6$$
$$x - 2y \leq 2$$
$$x \geq 0$$
$$y \geq 0$$
[4]

2. Given the regression equations: $2x - 7y + 2 = 0$ and $4y - 5x + 3 = 0$. Find the means of x and y. If the variance of x is 4, find the coefficient of correlation r and the variance of y. [6]