ISC CLASS XII MATHEMATICS (THEORY)

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. Candidates are required to attempt all questions from **Section A** and **EITHER Section B OR Section C**.
2. All working, including rough work, must be clearly shown and must be done on the same sheet as the rest of the answer. Omission of essential working will result in loss of marks.
3. The intended marks for questions or parts of questions are given in brackets [ ].

SECTION A (Compulsory - 65 Marks)

All questions in this section are compulsory.

Question 1 (10 × 1 Mark = 10 Marks)

Answer the following questions.

1. Let R be a relation on the set $\mathbb{Z}$ of integers defined by $(a, b) \in R$ if and only if $a - b$ is divisible by 3. State the number of distinct equivalence classes of R. [1]
2. If A is a square matrix of order 3 and $|A| = -5$, find the value of $|\text{adj}(A^2)|$. [1]
3. Evaluate: $\tan\left(2\tan^{-1}\left(\frac{1}{3}\right)\right)$. [1]
4. Find the value of k for which the function $f(x) = \begin{cases} \frac{1-\cos(kx)}{x^2} & \text{if } x \neq 0 \\ \frac{1}{2} & \text{if } x = 0 \end{cases}$ is continuous at $x = 0$. [1]
5. Find the order and degree of the differential equation: $\left(\frac{d^3y}{dx^3}\right)^2 - 5\frac{d^2y}{dx^2} + \sqrt{\frac{dy}{dx} + 1} = 0$. [1]
6. Evaluate: $\int_0^1 \frac{dx}{1+x^2}$. [1]
7. Prove that the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = |x - 1|$ is neither one-one nor onto. [1]
8. Using the first derivative test, find the local maxima/minima for $f(x) = x(x - 1)^2$. [1]
9. If $P(A) = 0.6$, $P(B) = 0.4$, and $P(A \cap B) = 0.24$. Are A and B independent events? Justify. [1]
10. A random variable X has the following probability distribution:

X	0	1	2
$P(X)$	0.3	k	0.2

Find the value of k. [1]

Question 2 (3 × 2 Marks = 6 Marks)

Answer the following questions.

1. If $y = (\cos x)^x + x^y$, find $\frac{dy}{dx}$. [2]
2. Verify Lagrange's Mean Value Theorem for the function $f(x) = x^2 - 4x - 3$ on the interval $[1, 4]$. [2]
3. If A and B are two independent events such that $P(A \cap B') = \frac{1}{6}$ and $P(A' \cap B) = \frac{1}{3}$, find $P(A)$ and $P(B)$. [2]

Question 3 (4 × 4 Marks = 16 Marks)

Answer the following questions.

1. If $x = a(\theta - \sin\theta)$ and $y = a(1 + \cos\theta)$, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{2}$. [4]
2. Find the equation of the tangent and normal to the curve $x^2 + y^2 = 25$ at the point $(3, -4)$. [4]
3. Using the properties of definite integrals, evaluate:

   $$\int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$$

   [4]
4. The two rows of a matrix A are R_1 and R_2. If $R_1 \leftrightarrow R_2$ gives matrix B, prove that $|B| = -|A|$ using properties of determinants. Hence, if

   $$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$$

   evaluate $\begin{vmatrix} a & b & c \\ 1 & 1 & 1 \\ a^3 & b^3 & c^3 \end{vmatrix}$. [4]

Question 4 (3 × 6 Marks = 18 Marks)

Answer the following questions.

1. Show that the semi-vertical angle of a cone of maximum volume and given slant height is $\tan^{-1}(\sqrt{2})$. [6]
2. Evaluate: $\int \frac{3x-1}{(x-1)(x-2)(x-3)} dx$. [6]
3. Solve the differential equation: $\frac{dy}{dx} + y \cot x = 2x + x^2 \cot x$. [6]

Question 5 (15 Marks)

Answer the following questions.

1. **(a)** Consider the function $f : \mathbb{R} \to [-1, 1]$ defined by $f(x) = \sin x$. Show that the relation f is not invertible. What modification to the domain of f is required to make it invertible? [5] **(b)** Solve the system of linear equations using the matrix inverse method: [6]

$$\begin{aligned} x + 2y - z &= 1 \\ 3x + 8y + 2z &= 5 \\ 4x + 9y - z &= 4 \end{aligned}$$

2. In a multiple-choice examination with three possible answers for each of the five questions, what is the probability that a candidate will get four or more correct answers by guessing? [4]

SECTION B (15 Marks)

Answer all questions from this section. (Unit V: Vectors - 5 Marks; Unit VI: 3D Geometry - 6 Marks; Unit VII: Applications of Integrals - 4 Marks)

Question 6 (5 Marks)

Answer the following questions.

1. **(a)** Find the projection of the vector $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$. [2] **(b)** Find a vector of magnitude 9 which is perpendicular to both the vectors $\vec{a} = 4\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = -2\hat{i} + \hat{j} - 2\hat{k}$. [3]

Question 7 (10 Marks)

Answer the following questions.

1. Find the equation of the plane passing through the intersection of the planes $x + 2y + 3z - 4 = 0$ and $2x + y - z + 5 = 0$ and which is perpendicular to the plane $5x + 3y + 6z + 8 = 0$. [6]
2. Using integration, find the area of the region bounded by the circle $x^2 + y^2 = 16$ and the parabola $y^2 = 6x$. [4]

SECTION C (15 Marks)

Answer all questions from this section. (Unit VIII: Application of Calculus - 5 Marks; Unit IX: Linear Regression - 6 Marks; Unit X: Linear Programming - 4 Marks)

Question 8 (5 Marks)

Answer the following questions.

1. A manufacturing company determines that the marginal cost for its product is given by $MC = 6x + 5$ and the marginal revenue is $MR = 10$. The fixed cost is Rs. 10. Find the total profit function $P(x)$ and determine the number of units x that maximizes the profit. Also, find the maximum profit. [5]

Question 9 (10 Marks)

Answer the following questions.

1. Solve the following Linear Programming Problem graphically: Maximize $Z = x + 2y$ Subject to the constraints:

$$\begin{aligned} x + y &\leq 9 \\ 2x + y &\geq 4 \\ x + 2y &\geq 6 \\ x, y &\geq 0 \end{aligned}$$

[4]

2. The following data gives the marks obtained by 10 students in Mathematics (x) and Physics (y):

x	35	40	30	50	45	25	60	55	42	38
y	40	50	35	55	60	30	65	58	45	40

Find the regression line of y on x and estimate the marks in Physics of a student who scored 48 marks in Mathematics. [6]