ISC CLASS XII MATHEMATICS (TEST PAPER 18) - SET 18

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. Candidates are required to attempt all questions from **Section A** and **EITHER Section B OR Section C**.
2. All working, including rough work, must be clearly shown. Omission of essential working will result in loss of marks.
3. The maximum mark for any single question is 6.
4. The intended marks for questions or parts of questions are given in brackets [ ].

SECTION A (Compulsory - 65 Marks)

All questions in this section are compulsory. (R&F: 10, Algebra: 10, Calculus: 32, Probability: 13)

Question 1 (10 × 1 Mark = 10 Marks)

Answer the following questions.

1. Let $*$ be a binary operation on $\mathbb{R}$ defined by $a * b = a + b - 1$. Find the inverse of the element 3. [1]
2. Evaluate: $\cos\left(\tan^{-1}\frac{3}{4}\right)$. [1]
3. State the range of the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = |x|$. [1]
4. State the value of $\det(\text{adj}(A))$ if A is a 3×3 matrix and $\det(A) = 5$. [1]
5. Find $\frac{dy}{dx}$ if $x^2 + y^2 = 25$. [1]
6. Write the value of $\int_0^2 x^2 dx$. [1]
7. Check if the differential equation $\frac{dy}{dx} = \frac{x^2+y^2}{x^2}$ is homogeneous. [1]
8. Determine the value of k for which $f(x) = \begin{cases} \frac{\sin 3x}{x} & \text{if } x \neq 0 \\ k & \text{if } x = 0 \end{cases}$ is continuous at $x = 0$. [1]
9. If A and B are independent events, simplify $P(A \cap B')$ in terms of $P(A)$ and $P(B)$. [1]
10. State whether the relation R defined on $\mathbb{Z}$ by aRb if $a - b$ is divisible by 3 is symmetric. [1]

Question 2 (3 × 2 Marks = 6 Marks)

Answer the following questions.

1. If $x = a(\theta - \sin\theta)$ and $y = a(1 - \cos\theta)$, find $\frac{dy}{dx}$. [2]
2. The perimeter of a square is increasing at the rate of 4 cm/s. Find the rate at which its area is increasing when the side length is 8 cm. [2]
3. Three coins are tossed. Find the probability of getting exactly two heads, given that the first toss is a head. [2]

Question 3 (4×4 Marks = 16 Marks)

Answer the following questions.

1. Evaluate: $\int x^2 \log x dx$. [4]
2. Find the particular solution of the differential equation: $\frac{dy}{dx} + y \cos x = \sin x \cos x$, given $y(0) = 1$. [4]
3. Verify Lagrange's Mean Value Theorem for the function $f(x) = x^2 + 2x + 3$ on the interval $[4, 6]$. [4]
4. If $A = \begin{pmatrix} 3 & 1 \\ 7 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 6 \\ 2 & 8 \end{pmatrix}$, find the matrix X such that $AX = B$. [4]

Question 4 (3×6 Marks = 18 Marks)

Answer the following questions.

1. Find the coordinates of the point on the curve $y = x^2 + 7x + 2$ which is closest to the origin. [6]
2. Evaluate: $\int \frac{dx}{\sqrt{x^2+4x+13}}$. [6]
3. Solve the following system of linear equations using the matrix method: [6]

$$\begin{aligned} x + 2y - 3z &= 6 \\ 2x - y + z &= 2 \\ 4x - 2y + 3z &= 4 \end{aligned}$$

Question 5 (15 Marks)

Answer the following questions.

1. **(a)** Prove that: $\cos^{-1} x + \cos^{-1} y = \cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right)$, where $x, y \in [-1, 1]$ and $x + y \geq 0$. [6] **(b)** The probability of a successful hit by a gun is 0.1. Find the minimum number of shots required so that the probability of at least one hit is greater than 0.5. [6] **(c)** Find the general solution of the differential equation: $\frac{dy}{dx} = \frac{x^2+3y^2}{3x^2+y^2}$. [3]

SECTION B (Optional - 15 Marks)

Answer all questions from this section. (Unit V: Vectors - 5 Marks; Unit VI: 3D Geometry - 6 Marks; Unit VII: Applications of Integrals - 4 Marks)

Question 6 (5 Marks)

Answer the following questions.

1. Find the area of the parallelogram whose diagonals are $\vec{d_1} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{d_2} = \hat{i} + 3\hat{j} - \hat{k}$. [2]
2. Show that the vectors $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = -2\hat{i} + 3\hat{j} + 4\hat{k}$, and $\vec{c} = -3\hat{j} + 10\hat{k}$ are coplanar. [3]

Question 7 (10 Marks)

Answer the following questions.

1. Find the image of the point $(1, 6, 3)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. [6]
2. Using integration, find the area bounded by the parabola $y^2 = 4x$ and the line $y = x$. [4]

SECTION C (Optional - 15 Marks)

Answer all questions from this section. (Unit VIII: Application of Calculus - 5 Marks; Unit IX: Linear Regression - 6 Marks; Unit X: Linear Programming - 4 Marks)

Question 8 (5 Marks)

Answer the following question.

1. The total cost function is given by $C(x) = 2x^2 + 3x + 1$. Find the marginal cost (MC) and the marginal revenue (MR) when $x = 1$, if the total revenue function is $R(x) = 10x - x^2$. Hence, comment on the profit level. [5]

Question 9 (10 Marks)

Answer the following questions.

1. Solve the following Linear Programming Problem graphically: Minimize $Z = 3x + 5y$ Subject to the constraints:

$$x + 3y \geq 3$$
$$x + y \geq 2$$
$$x, y \geq 0$$

   [4]

2. The two lines of regression are $y = 2x + 1$ and $x = 0.5y + 3$. Find the means of x and y. Hence, find the coefficient of correlation r. [6]