PRACTICE QUESTION PAPER - XVII
CLASS XII - MATHEMATICS (041)

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper contains **38** questions. All questions are compulsory.
2. The question paper is divided into FIVE Sections – A, B, C, D and E.
3. Section **A** comprises of **20** questions of **1** mark each. (18 MCQs + 2 Assertion-Reasoning)
4. Section **B** comprises of **5** questions of **2** marks each.
5. Section **C** comprises of **6** questions of **3** marks each.
6. Section **D** comprises of **4** questions of **5** marks each.
7. Section **E** comprises of **3** Case Study Based Questions of **4** marks each.
8. There is no overall choice in the question paper. However, an internal choice has been provided in **2** questions in Section B, **3** questions in Section C, **2** questions in Section D and **2** questions in Section E (in the sub-parts).
9. Use of calculators is **not** permitted.

SECTION A (20 Marks)

This section comprises **20** *questions of* **1** *mark each. Questions 1-18 are Multiple Choice Questions (MCQs) and questions 19-20 are Assertion-Reason based questions.*

Multiple Choice Questions (MCQs) and Assertion-Reason Questions (Combined Enumeration)

1. The binary operation $*$ on $\mathbb{Z}$ defined by $a * b = a^2 + b^2$ is:
   (a) Commutative and Associative
   (b) Associative but not Commutative
   (c) Commutative but not Associative
   (d) Neither Commutative nor Associative
2. The function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \lfloor x \rfloor$ (greatest integer function) is:
   (a) One-one
   (b) Onto
   (c) Bijective
   (d) Neither one-one nor onto
3. The value of $\tan\left(\frac{1}{2}\cos^{-1}\frac{7}{25}\right)$ is:
   (a) 3/4
   (b) 1/7
   (c) 4/3
   (d) 1/5
4. The range of $\sec^{-1} x$ is:
   (a) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
   (b) $[0, \pi]$
   (c) $[0, \pi] - \left\{\frac{\pi}{2}\right\}$
   (d) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

5. If A is a square matrix of order n, then $|adj(\text{adj } A)|$ is equal to:

   (a) $|A|^{n-1}$

   (b) $|A|^{(n-1)^2}$

   (c) $|A|^{n^2}$

   (d) $|A|^n$

6. If $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, then the matrix product AB is:

   (a) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

   (b) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

   (c) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

   (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

7. If the matrix $A = \begin{bmatrix} 2 & k \\ 3 & 4 \end{bmatrix}$ is singular, then the value of k is:

   (a) $3/8$

   (b) $8/3$

   (c) $-8/3$

   (d) 4

8. If A is an invertible matrix of order 2, then $\det(A^{-1})$ is equal to:

   (a) $\det(A)$

   (b) $\frac{1}{\det(A)}$

   (c) 1

   (d) 0

9. If $y = \log(\tan x)$, then $\frac{dy}{dx}$ is:

   (a) $\sec x \tan x$

   (b) $\frac{\sec^2 x}{x}$

   (c) $\frac{2}{\sin 2x}$

   (d) $\sec^2 x$

10. The function $f(x) = x^3 - 6x^2 + 15x - 8$ is:

    (a) Strictly decreasing on $\mathbb{R}$

    (b) Strictly increasing on $\mathbb{R}$

    (c) Decreasing on $(0, 4)$

    (d) Increasing on $(0, 4)$

11. $\int e^x(\sec x + \sec x \tan x)\, dx$ is equal to:

    (a) $e^x \tan x + C$

    (b) $e^x \sec x + C$

    (c) $e^x \cot x + C$

(d) $e^x \csc x + C$

12. The value of $\int_0^\pi \frac{dx}{1+\sin x}$ is:

    (a) 0

    (b) 1

    (c) 2

    (d) $\pi/2$

13. The particular solution of $\frac{dy}{dx} = 4x^3y$, where $y(0) = 1$, is:

    (a) $y = e^{x^4}$

    (b) $y = e^{4x^3}$

    (c) $y = x^4 + 1$

    (d) $y = \log(x^4 + 1)$

14. If the vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 3, |\vec{b}| = 2\sqrt{2}$ and $|\vec{a} \times \vec{b}| = 6$, then the angle between $\vec{a}$ and $\vec{b}$ is:

    (a) $\pi/4$

    (b) $\pi/6$

    (c) $\pi/3$

    (d) $\pi/2$

15. If $\vec{a}$ is a non-zero vector of magnitude a and λ is a non-zero scalar, then $\lambda\vec{a}$ is a unit vector if:

    (a) $\lambda = 1$

    (b) $\lambda = -1$

    (c) $a = |\lambda|$

    (d) $a = 1/|\lambda|$

16. The intercepts made by the plane $x - 2y + 3z = 6$ on the coordinate axes are:

    (a) $6, -3, 2$

    (b) $1, -2, 3$

    (c) $6, 3, 2$

    (d) $1, -1/2, 1/3$

17. The angle between the two lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ is:

    (a) $\cos^{-1}\left(\frac{10}{\sqrt{14}\sqrt{29}}\right)$

    (b) $\cos^{-1}\left(\frac{20}{29}\right)$

    (c) $\cos^{-1}\left(\frac{20}{\sqrt{14}\sqrt{29}}\right)$

    (d) $\cos^{-1}\left(\frac{10}{29}\right)$

18. The feasible region for an LPP is shown in the figure. If the objective function is $Z = 3x - 4y$, the minimum value of Z occurs at:

    (a) $(0, 0)$

    (b) $(0, 8)$

    (c) $(5, 0)$

    (d) $(4, 10)$

Assertion-Reasoning Based Questions

In questions 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer from the following options:

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. **Assertion (A):** For any two events A and B, $P(A' \cup B') = 1 - P(A \cap B)$. **Reason (R):** $A' \cup B' = (A \cap B)'$ by De Morgan's Law, and $P(E') = 1 - P(E)$.

20. **Assertion (A):** The derivative of x^x with respect to x is $x^x(1 + \log x)$. **Reason (R):** $\frac{d}{dx}(x^n) = nx^{n-1}$ for any real number n.

SECTION B (10 Marks)

This section comprises **5** *questions of* **2** *marks each.*

21. Find $\frac{dy}{dx}$ if $x\sqrt{1+y} + y\sqrt{1+x} = 0$, where $x \neq y$.

22. Find the vector equation of the line passing through the point $(5, 2, -4)$ and parallel to the vector $3\hat{i} + 2\hat{j} - 8\hat{k}$.

OR

If the dot product of two vectors is 12 and the magnitude of their cross product is $6\sqrt{3}$, find the angle between the two vectors.

23. Evaluate $\int e^x \left(\frac{x-3}{(x-1)^3} \right) dx$.

OR

If $y = \cos^{-1} x$, find $\frac{d^2y}{dx^2}$ in terms of y alone.

24. Find the values of x and y if $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$.

25. A pair of dice is thrown. Find the probability of getting a doublet or a sum of 8.

SECTION C (18 Marks)

This section comprises **6** *questions of* **3** *marks each.*

26. If $f(x) = \frac{4x+3}{6x-4}, x \neq \frac{2}{3}$, show that $f \circ f(x) = x$. Hence, find $f^{-1}(x)$.

27. Evaluate $\int \frac{x^2+1}{(x^2+2)(x^2+3)} dx$.

OR

Evaluate $\int \frac{dx}{\sin x + \cos x}$.

28. Find the particular solution of the differential equation $\frac{dy}{dx} = 1 + x + y + xy$, given that $y(0) = 0$.

OR

Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$ which is parallel to the line $2x - y + 9 = 0$.

29. Find the vector equation of the plane passing through the intersection of the planes $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 6$ and $\vec{r} \cdot (2\hat{i} + 3\hat{j} + 4\hat{k}) = -5$ and the point $(1, 1, 1)$.

OR

Show that the lines $\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5}$ and $\frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3}$ are coplanar.

30. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ and $A + A^T = I$, then find the value of α.

31. Determine the maximum value of $Z = 11x + 7y$ subject to $x \leq 3, y \leq 2, x \geq 0, y \geq 0$.

SECTION D (20 Marks)

This section comprises **4** *questions of* **5** *marks each.*

32. Using integration, find the area of the region bounded by the curves $y = |x-1|$ and $y = 1$.

OR

Evaluate $\int_0^{\pi/4} \frac{\sin x + \cos x}{9 + 16\sin 2x}\, dx$.

33. Use matrix method to solve the following system of equations: $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$, $\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$, $\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$.

34. Find the maximum volume of an inverted cone with semi-vertical angle α and slant height L.

OR

Evaluate $\int \frac{x^2+1}{(x-1)^2(x+3)}\, dx$.

35. Find the coordinates of the foot of the perpendicular drawn from the point $A(1, 8, 4)$ to the line joining $B(0, -1, 3)$ and $C(2, -3, -1)$.

SECTION E (12 Marks)

This section comprises **3** *case study based questions of* **4** *marks each.*

36. **Case Study 1: Area Minimization**

A rectangular sheet of metal is 12 cm wide and 20 cm long. A square is cut from each corner of the sheet, and the remaining material is folded up to form an open box with a square base. Let x be the side length of the square cut out.

Based on the given information, answer the following questions:

(a) Find the dimensions of the base of the open box in terms of x. (1 Mark)

(b) Express the volume V of the box as a function of x. (3 Marks)

OR

(c) Find the value of x for which the volume of the box is maximum. (3 Marks)

37. **Case Study 2: Bayes' Theorem in Production**

A factory has three machines M_1, M_2, M_3 which produce $30\%, 40\%, 30\%$ of the total production, respectively. It is known that 2% of the products from M_1, 1% from M_2, and 3% from M_3 are defective. A product is selected at random and found to be defective.

Based on the given information, answer the following questions:

(a) Find the probability that a product is defective, given it came from machine M_2. (1 Mark)

(b) Find the probability that a randomly selected product is defective. (3 Marks)

OR

(c) Find the probability that the defective product was produced by machine M_1. (3 Marks)

38. **Case Study 3: Traffic Lights and Intersections**

Two lines of sight from a traffic control tower to two vehicles are modeled by the lines L_1 and L_2 passing through the origin.

$$L_1 : \frac{x}{1} = \frac{y}{1} = \frac{z}{-2}$$
$$L_2 : \frac{x}{2} = \frac{y}{-1} = \frac{z}{1}$$

The horizontal ground is modelled by the plane $z = 0$.

Based on the given information, answer the following questions:

(a) Write the direction vectors of the lines L_1 and L_2. (1 Mark)

(b) Find the angle between the two lines of sight L_1 and L_2. (3 Marks)

OR

(c) Find the coordinate of the point where the line L_1 intersects the plane $x - y + 2z = 5$. (3 Marks)