PRACTICE QUESTION PAPER - XIV
CLASS XII - MATHEMATICS (041)

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper contains **38** questions. All questions are compulsory.
2. The question paper is divided into FIVE Sections – A, B, C, D and E.
3. Section **A** comprises of **20** questions of **1** mark each. (18 MCQs + 2 Assertion-Reasoning)
4. Section **B** comprises of **5** questions of **2** marks each.
5. Section **C** comprises of **6** questions of **3** marks each.
6. Section **D** comprises of **4** questions of **5** marks each.
7. Section **E** comprises of **3** Case Study Based Questions of **4** marks each.
8. There is no overall choice in the question paper. However, an internal choice has been provided in **2** questions in Section B, **3** questions in Section C, **2** questions in Section D and **2** questions in Section E (in the sub-parts).
9. Use of calculators is **not** permitted.

SECTION A (20 Marks)

This section comprises **20** *questions of* **1** *mark each. Questions 1-18 are Multiple Choice Questions (MCQs) and questions 19-20 are Assertion-Reason based questions.*

Multiple Choice Questions (MCQs) and Assertion-Reason Questions (Combined Enumeration)

1. The relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$ is:
   (a) Reflexive but not symmetric
   (b) Symmetric but not transitive
   (c) Transitive but not reflexive
   (d) An equivalence relation
2. If $f(x) = x^2$ and $g(x) = 2x + 1$ are two functions, then the composite function $(g \circ f)(x)$ is:
   (a) $(2x + 1)^2$
   (b) $2x^2 + 1$
   (c) $4x^2 + 4x + 1$
   (d) $x^2 + 2x + 1$
3. The value of $\cos(\sec^{-1} x + \csc^{-1} x)$, $|x| \geq 1$, is:
   (a) 1
   (b) 0
   (c) -1
   (d) $\pi/2$
4. If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$, then $\cos^{-1} x + \cos^{-1} y$ is equal to:
   (a) $\frac{\pi}{3}$
   (b) $\frac{\pi}{6}$
   (c) $\frac{4\pi}{3}$
   (d) $\frac{\pi}{2}$

5. If matrix A has order $3 \times n$ and matrix B has order $m \times 5$, and both AB and BA are defined, then the values of m and n are:

   (a) $m = 3, n = 5$

   (b) $m = 5, n = 3$

   (c) $m = 3, n = 3$

   (d) $m = 5, n = 5$

6. If $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, then A^{10} is equal to:

   (a) $2^{10}A$

   (b) 2^9A

   (c) 2^8A

   (d) $10A$

7. If A is a square matrix of order 3 such that $A(\text{adj } A) = 10I$, then $|A|$ is:

   (a) 1

   (b) 10

   (c) 100

   (d) 1000

8. For what value of k is the area of the triangle with vertices $(k, 0), (4, 0), (0, 2)$ equal to 4 square units?

   (a) 0

   (b) 8

   (c) 0 or 8

   (d) -8

9. If $y = \log(\sqrt{\frac{1+x}{1-x}})$, then $\frac{dy}{dx}$ is:

   (a) $\frac{1}{1-x^2}$

   (b) $\frac{1}{x^2-1}$

   (c) $\frac{1}{\sqrt{1-x^2}}$

   (d) $\frac{x}{1-x^2}$

10. The maximum value of $\sin x + \cos x$ is:

    (a) 1

    (b) 2

    (c) $\sqrt{2}$

    (d) $\frac{1}{\sqrt{2}}$

11. $\int \frac{1}{\sqrt{x+a}+\sqrt{x+b}}\, dx$ is proportional to:

    (a) $\frac{(x+a)^{3/2}-(x+b)^{3/2}}{a-b}$

    (b) $\frac{(x+a)^{3/2}+(x+b)^{3/2}}{a-b}$

    (c) $(x + a)^{3/2} - (x + b)^{3/2}$

    (d) $\frac{(x+a)^{3/2}}{a-b}$

12. The value of $\int_{-\pi/2}^{\pi/2} \sin^7 x \, dx$ is:

(a) 1

(b) -1

(c) 0

(d) $\pi/2$

13. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$ is:

(a) 3

(b) 2

(c) 1

(d) Not defined

14. The angle between the vector $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and the positive direction of the z-axis is:

(a) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(b) $\cos^{-1}\left(\frac{1}{3}\right)$

(c) $\pi/4$

(d) $\pi/3$

15. If $\vec{a}, \vec{b}, \vec{c}$ are three unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ is:

(a) 3

(b) -3

(c) $3/2$

(d) $-3/2$

16. The line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ is parallel to the plane:

(a) $2x + 3y + 4z = 0$

(b) $3x + 4y + 5z = 0$

(c) $3x + 4y - 5z = 0$

(d) $6x + 8y - 10z = 0$

17. The perpendicular distance from the origin to the plane $x - 2y + 2z = 9$ is:

(a) 3

(b) 9

(c) 1

(d) $\sqrt{9}$

18. The maximum value of $Z = 3x + 4y$ subject to $x + y \leq 4, x \geq 0, y \geq 0$ is obtained at the point:

(a) $(0, 4)$

(b) $(4, 0)$

(c) $(0, 0)$

(d) $(1, 3)$

Assertion-Reasoning Based Questions

In questions 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer from the following options:

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. **Assertion (A):** If A and B are events such that $P(A|B) = P(B|A)$, then $P(A) = P(B)$. **Reason (R):** $P(A|B) = \frac{P(A\cap B)}{P(B)}$ and $P(B|A) = \frac{P(A\cap B)}{P(A)}$.

20. **Assertion (A):** $\frac{d}{dx}(\sin x^2) = 2x\cos x^2$. **Reason (R):** The differentiation is done using the Chain Rule, $\frac{d}{dx}f(g(x)) = f'(g(x))\cdot g'(x)$.

SECTION B (10 Marks)

This section comprises **5** *questions of* **2** *marks each.*

21. Find $\frac{dy}{dx}$ if $y = \sin^{-1}\left(\frac{2^x}{1+4^x}\right)$.

22. Find the angle between the vectors $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$.

OR

Find the projection of the vector $7\hat{i} + \hat{j} - 4\hat{k}$ on the vector $2\hat{i} + 6\hat{j} + 3\hat{k}$.

23. Evaluate $\int \frac{1}{\sqrt{1-4x^2}}\,dx$.

OR

Find the points on the curve $y = x^2 - 2x + 3$ where the tangent is parallel to the x-axis.

24. For a 2×2 matrix $A = [a_{ij}]$ whose elements are given by $a_{ij} = \frac{(i+j)^2}{2}$, construct the matrix A.

25. Given $P(A) = 0.3, P(B) = 0.5$ and $P(A\cap B) = 0.1$. Find the probability that neither A nor B occurs.

SECTION C (18 Marks)

This section comprises **6** *questions of* **3** *marks each.*

26. Show that $f : \mathbb{R} - \{3\} \to \mathbb{R} - \{1\}$ defined by $f(x) = \frac{x-2}{x-3}$ is a bijective function.

27. Evaluate $\int \frac{e^x}{x}(x\log x + 1)\,dx$.

OR

Evaluate $\int \frac{x-1}{\sqrt{x^2-1}}\,dx$.

28. Solve the differential equation $x\frac{dy}{dx} - y = \sqrt{x^2+y^2}$.

OR

Find the intervals in which the function $f(x) = \sin x - \cos x$, $0 < x < 2\pi$ is strictly increasing.

29. Find the equation of the plane passing through the point $(1, 1, -1)$ and perpendicular to the line joining the points $A(4, 3, 2)$ and $B(1, -1, 0)$.

OR

Find the vector equation of the line passing through the points $(-2, 0, 3)$ and $(3, 5, -2)$.

30. If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$, find a value of k such that $A^2 = kA - 2I$.

31. Solve the following Linear Programming Problem graphically: Minimize $Z = 3x + 5y$ subject to $x + 3y \geq 3$, $x + y \geq 2$, $x, y \geq 0$.

SECTION D (20 Marks)

*This section comprises **4** questions of **5** marks each.*

33. Using integration, find the area of the region bounded by the curves $y^2 = 9x$ and $x^2 = 9y$.

OR

Evaluate $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x}\, dx$.

34. Obtain the inverse of the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ using elementary row operations.

35. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$.

OR

Evaluate $\int \frac{\sin x \cos x}{\cos^2 x + 3\cos x + 2}\, dx$.

36. Find the equation of the line passing through the point $P(4, 6, 2)$ and the point of intersection of the line $\frac{x-1}{3} = \frac{y}{2} = \frac{z+1}{7}$ and the plane $x + y - z = 8$.

SECTION E (12 Marks)

*This section comprises **3** case study based questions of **4** marks each.*

37. **Case Study 1: Velocity, Acceleration, and Distance**

A particle moves such that its position in metres at time t seconds is given by $s(t) = t^3 - 6t^2 + 9t + 10$. The velocity $v(t)$ and acceleration $a(t)$ are given by $v(t) = \frac{ds}{dt}$ and $a(t) = \frac{dv}{dt}$.

Based on the given information, answer the following questions:

(a) Find the velocity function $v(t)$ of the particle. (1 Mark)

(b) Find the time interval when the particle is moving in the positive direction (i.e., $v(t) > 0$). (3 Marks)

OR

(c) Find the acceleration of the particle when its velocity is zero. (3 Marks)

38. **Case Study 2: Distribution of Coloured Balls**

A bag contains 4 red and 6 black balls. Three balls are drawn at random with replacement. Let X be the number of red balls drawn.

Based on the given information, answer the following questions:

(a) Find the probability of drawing a red ball in a single draw. (1 Mark)

(b) Determine the probability distribution of X. (3 Marks)

OR

(c) Find the probability of drawing at most 2 red balls. (3 Marks)

39. **Case Study 3: Angle Between Lines in a Structure**

In a structural design, two straight support beams are modelled as lines L_1 and L_2. Their equations are:

$$L_1 : \frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$$

$$L_2 : \frac{x}{-1} = \frac{y}{1} = \frac{z-1}{2}$$

Based on the given information, answer the following questions:

(a) Write the direction ratios of the beam L_1 and L_2. (1 Mark)

(b) Find the vector representing the direction of the line L_2. (3 Marks)

OR

(c) Calculate the acute angle between the two beams L_1 and L_2. (3 Marks)