PRACTICE QUESTION PAPER - XV
CLASS XII - MATHEMATICS (041)

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper contains **38** questions. All questions are compulsory.
2. The question paper is divided into FIVE Sections – A, B, C, D and E.
3. Section **A** comprises of **20** questions of **1** mark each. (18 MCQs + 2 Assertion-Reasoning)
4. Section **B** comprises of **5** questions of **2** marks each.
5. Section **C** comprises of **6** questions of **3** marks each.
6. Section **D** comprises of **4** questions of **5** marks each.
7. Section **E** comprises of **3** Case Study Based Questions of **4** marks each.
8. There is no overall choice in the question paper. However, an internal choice has been provided in **2** questions in Section B, **3** questions in Section C, **2** questions in Section D and **2** questions in Section E (in the sub-parts).
9. Use of calculators is **not** permitted.

SECTION A (20 Marks)

This section comprises **20** *questions of* **1** *mark each. Questions 1-18 are Multiple Choice Questions (MCQs) and questions 19-20 are Assertion-Reason based questions.*

Multiple Choice Questions (MCQs) and Assertion-Reason Questions (Combined Enumeration)

1. The maximum number of equivalence relations on the set $A = \{1, 2, 3\}$ is:
   (a) 1
   (b) 2
   (c) 3
   (d) 5
2. If $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = \frac{x^2}{1+x^2}$, then the range of the function f is:
   (a) $[0, 1]$
   (b) $[0, 1)$
   (c) $(0, 1)$
   (d) $(0, 1]$
3. If $\tan^{-1} x = \frac{\pi}{10}$, then the value of $\cot^{-1} x$ is:
   (a) $\frac{2\pi}{5}$
   (b) $\frac{3\pi}{5}$
   (c) $\frac{4\pi}{5}$
   (d) $\frac{3\pi}{10}$
4. The principal value of $\sin^{-1}\left(\sin \frac{3\pi}{5}\right)$ is:
   (a) $\frac{3\pi}{5}$
   (b) $\frac{2\pi}{5}$
   (c) $-\frac{3\pi}{5}$

(d) $-\frac{2\pi}{5}$

5. If A is a matrix of order 3×4, and B is a matrix of order 4×3, then the order of BA is:
   (a) 3×3
   (b) 4×4
   (c) 3×4
   (d) 4×3
6. If A is a square matrix such that $A^2 = I$, then $(A - I)^3 + (A + I)^3 - 7A$ is equal to:
   (a) A
   (b) I
   (c) $3A$
   (d) $5A$
7. If A is a non-singular matrix of order 3 and $|A| = 5$, then $|A^{-1}|$ is:
   (a) 5
   (b) 1/5
   (c) 25
   (d) -5
8. For which value of x is the matrix $\begin{bmatrix} 3-2x & x+1 \\ 2 & 4 \end{bmatrix}$ singular?
   (a) 1
   (b) 5
   (c) 3
   (d) 2
9. If $y = \cos(\log x) + \sin(\log x)$, then $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx}$ is equal to:
   (a) y
   (b) $-y$
   (c) x
   (d) 0
10. The rate of change of the area of a circle with respect to its circumference is:
   (a) $2\pi r$
   (b) r
   (c) $2r$
   (d) πr^2
11. $\int e^x \left(\frac{1-\sin x}{1-\cos x}\right) dx$ is equal to:
   (a) $e^x \tan(x/2) + C$
   (b) $e^x \cot(x/2) + C$
   (c) $-e^x \tan(x/2) + C$
   (d) $-e^x \cot(x/2) + C$
12. The value of $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$ is:
   (a) $\pi/2$

(b) $\pi/4$

(c) π

(d) 1

13. The general solution of the differential equation $\log\left(\frac{dy}{dx}\right) = ax + by$ is:

(a) $ae^{ax} + be^{by} = C$

(b) $\frac{1}{a}e^{ax} + \frac{1}{b}e^{-by} = C$

(c) $\frac{1}{a}e^{ax} - \frac{1}{b}e^{-by} = C$

(d) $e^{ax} - e^{-by} = C$

14. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then the vectors $\vec{a}$ and $\vec{b}$ are:

(a) Parallel

(b) Perpendicular

(c) Unit vectors

(d) Equal in magnitude

15. If $\vec{a} = 3\hat{i} - \hat{j} + 4\hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - 2\hat{k}$, then the magnitude of the vector $\vec{a} \times \vec{b}$ is:

(a) 8

(b) 10

(c) $\sqrt{108}$

(d) $\sqrt{126}$

16. The shortest distance between the x-axis and the z-axis is:

(a) 0

(b) 1

(c) $\sqrt{2}$

(d) ∞

17. The direction cosines of the normal to the plane $2x + 3y - z = 4$ are:

(a) $\left(\frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{-1}{\sqrt{14}}\right)$

(b) $(2, 3, -1)$

(c) $\left(\frac{2}{4}, \frac{3}{4}, \frac{-1}{4}\right)$

(d) $\left(\frac{2}{\sqrt{6}}, \frac{3}{\sqrt{6}}, \frac{-1}{\sqrt{6}}\right)$

18. For the constraints $x + y \leq 4, x \geq 0, y \geq 0$, the maximum value of $Z = x + 2y$ is:

(a) 4

(b) 8

(c) 6

(d) 2

Assertion-Reasoning Based Questions

In questions 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer from the following options:

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. **Assertion (A):** If A and B are events such that $P(A) \neq 0$, and A is a subset of B, then $P(B|A) = 1$. **Reason (R):** If $A \subset B$, then $A \cap B = A$.

20. **Assertion (A):** The function $f(x) = \tan x - x$ is always increasing. **Reason (R):** $f'(x) = \sec^2 x - 1$, and $\sec^2 x - 1 \geq 0$ for all $x \in \mathbb{R}$.

SECTION B (10 Marks)

This section comprises **5** *questions of* **2** *marks each.*

21. Find $\frac{dy}{dx}$ if $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right)$.

22. Show that the points $A(1, 2, 7)$, $B(2, 6, 3)$ and $C(3, 10, -1)$ are collinear.

OR

If the position vector of point A is $\vec{a}$, and the position vector of point B is $\vec{b}$, find the position vector of a point C which divides AB internally in the ratio $2 : 3$.

23. Evaluate $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha}\, dx$.

OR

Find the slope of the normal to the curve $y = x^3 - x$ at $x = 2$.

24. If $A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$, find A^{-1}.

25. A die is thrown twice and the sum of the numbers appearing is observed to be 7. What is the conditional probability that the number 3 has appeared at least once?

SECTION C (18 Marks)

This section comprises **6** *questions of* **3** *marks each.*

26. Prove that $\sin^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{5}{13}\right) = \sin^{-1}\left(\frac{63}{65}\right)$.

27. Evaluate $\int \frac{x^2}{x^2+7}\, dx$.

OR

Evaluate $\int \frac{dx}{\sqrt{7-3x-2x^2}}$.

28. Find the particular solution of the differential equation $(x + 2y^2)\frac{dy}{dx} = y$, given that $y = 1$ when $x = 1$.

OR

Find the maximum and minimum value of $f(x) = (\sin x + \cos x)$ in the interval $[0, \pi]$.

29. Find the angle between the line $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ and the plane $10x + 2y - 11z = 3$.

OR

Find the direction cosines of the vector $\vec{a} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ and calculate the scalar component of this vector along the y-axis.

30. If $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, show that $(aI + bA)^3 = a^3 I + 3a^2 bA - 3ab^2 I - b^3 A$.

31. A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of Rs 17.50 per package on nuts and Rs 7.00 per

package on bolts. The machines A and B are available for a maximum of 12 hours each. Formulate the problem as a linear programming problem (LPP) to maximize his profit.

SECTION D (20 Marks)

This section comprises **4** *questions of* **5** *marks each.*

32. Using integration, find the area of the region bounded by the curve $y = \sqrt{x}$ and the lines $x = 2$ and $x = 4$.

OR

Evaluate $\int_0^{\pi} \frac{x}{1+\sin x}\, dx$.

33. Solve the system of linear equations using the matrix method: $2x + 3y + 3z = 5$, $x - 2y + z = -4$, $3x - y - 2z = 3$.

34. Show that the semi-vertical angle of a cone of maximum volume and given slant height is $\tan^{-1}(\sqrt{2})$.

OR

Evaluate $\int \frac{\sin x}{3\sin x + 4\cos x}\, dx$.

35. Find the equation of the plane passing through the intersection of the planes $2x + y + 3z - 2 = 0$ and $x - 2y + z = 6$ and parallel to the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$.

SECTION E (12 Marks)

This section comprises **3** *case study based questions of* **4** *marks each.*

36. **Case Study 1: Rate of Change in Science**

A chemical reaction rate is modelled by the concentration function $C(t) = ae^{-kt} + b$, where $C(t)$ is the concentration of the chemical at time t, and a, b, k are positive constants. The rate of change of concentration is $\frac{dC}{dt}$.

Based on the given information, answer the following questions:

(a) Find the initial concentration of the chemical (at $t = 0$). (1 Mark)

(b) Find the rate of change of concentration $\frac{dC}{dt}$. (3 Marks)

OR

(c) Show that the rate of change of concentration is proportional to $C(t) - b$. (3 Marks)

37. **Case Study 2: Reliability of a System**

A system of electronic components consists of two independent parts, A and B. The probability that component A works is 0.9 and the probability that component B works is 0.8. The system fails if both components A and B fail.

Based on the given information, answer the following questions:

(a) Find the probability that component A fails. (1 Mark)

(b) Find the probability that the system fails. (3 Marks)

OR

(c) Find the probability that the system works (i.e., at least one component works). (3 Marks)

38. **Case Study 3: Path of an Aeroplane and Plane Equation**

An aeroplane flies along the line $\vec{r} = (5\hat{i} - \hat{j} + \hat{k}) + \lambda(\hat{i} + \hat{j} - 2\hat{k})$. It passes through a specific air-traffic zone defined by the plane $x + y + z = 7$.

Based on the given information, answer the following questions:

(a) Write the coordinates of the fixed point through which the aeroplane passes. (1 Mark)

(b) Find the equation of the plane parallel to the given air-traffic zone plane and passing through the origin. (3 Marks)

OR

(c) Find the distance of the origin from the given air-traffic zone plane $x + y + z = 7$. (3 Marks)