PRACTICE QUESTION PAPER - XVI
CLASS XII - MATHEMATICS (041)

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper contains **38** questions. All questions are compulsory.
2. The question paper is divided into FIVE Sections – A, B, C, D and E.
3. Section **A** comprises of **20** questions of **1** mark each. (18 MCQs + 2 Assertion-Reasoning)
4. Section **B** comprises of **5** questions of **2** marks each.
5. Section **C** comprises of **6** questions of **3** marks each.
6. Section **D** comprises of **4** questions of **5** marks each.
7. Section **E** comprises of **3** Case Study Based Questions of **4** marks each.
8. There is no overall choice in the question paper. However, an internal choice has been provided in **2** questions in Section B, **3** questions in Section C, **2** questions in Section D and **2** questions in Section E (in the sub-parts).
9. Use of calculators is **not** permitted.

SECTION A (20 Marks)

This section comprises **20** *questions of* **1** *mark each. Questions 1-18 are Multiple Choice Questions (MCQs) and questions 19-20 are Assertion-Reason based questions.*

Multiple Choice Questions (MCQs) and Assertion-Reason Questions (Combined Enumeration)

1. Let $A = \{1, 2, 3\}$. The total number of non-empty subsets of A that are closed under the binary operation $*$ defined by $a * b = \min(a, b)$ is:
   (a) $2^3 - 1 = 7$
   (b) 3
   (c) 6
   (d) 7
2. If $f(x) = |x - 1|$, then the function $f : \mathbb{R} \to [0, \infty)$ is:
   (a) One-one
   (b) Onto
   (c) Bijective
   (d) Neither one-one nor onto
3. The value of $\tan^{-1}(2) + \tan^{-1}(3)$ is:
   (a) $\pi/4$
   (b) $\pi/2$
   (c) $3\pi/4$
   (d) π
4. The domain of the function $f(x) = \sin^{-1}(3x - 1)$ is:
   (a) $[0, 2/3]$
   (b) $[-1, 1]$
   (c) $[1/3, 2/3]$

(d) $[2/3, 1]$

5. If $A = \begin{bmatrix} x+y & 2 \\ 5 & x-y \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 \\ 5 & 1 \end{bmatrix}$ are such that $A = B$, then $x^2 + y^2$ is equal to:
   (a) 5
   (b) 2
   (c) 3
   (d) 1
6. If A is a square matrix, then the matrix $A + A^T$ is a:
   (a) Skew-symmetric matrix
   (b) Symmetric matrix
   (c) Orthogonal matrix
   (d) Unit matrix
7. If A is a square matrix of order 3 and $|A| = -2$, then the value of $|-A|$ is:
   (a) -2
   (b) 2
   (c) 8
   (d) -8
8. If A is a non-singular matrix, then $adj(A)$ is equal to:
   (a) $|A|A^{-1}$
   (b) $|A|I$
   (c) $A/|A|$
   (d) $|A|A$
9. If $x = at^2$ and $y = 2at$, then $\frac{dy}{dx}$ is:
   (a) t
   (b) $1/t$
   (c) $2/t$
   (d) $2t$
10. The slope of the tangent to the curve $y = x^3 - x^2 + 1$ at the point where $x = 1$ is:
    (a) 1
    (b) 2
    (c) -1
    (d) 0
11. $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} \, dx$ is equal to:
    (a) $\tan x + \cot x + C$
    (b) $\tan x - \cot x + C$
    (c) $-\tan x - \cot x + C$
    (d) $-\tan x + \cot x + C$
12. $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} \, dx$ is equal to:
    (a) 0

(b) $\pi/4$

(c) $\pi/2$

(d) π

13. The general solution of the differential equation $\frac{dy}{dx} = 2^{y-x}$ is:

(a) $2^x + 2^y = C$

(b) $2^{-x} + 2^{-y}\log 2 = C$

(c) $2^x - 2^{-y} = C$

(d) $2^x + 2^{-y}/\log 2 = C$

14. The two vectors $\hat{\imath} + 3\hat{\jmath} - 2\hat{k}$ and $2\hat{\imath} + \lambda\hat{\jmath} + 4\hat{k}$ are orthogonal if λ is:

(a) 2

(b) 0

(c) -2

(d) 1

15. If the magnitude of the scalar projection of $\vec{a}$ on $\vec{b}$ is 1, where $\vec{a} = 3\hat{\imath} - \hat{\jmath} - 5\hat{k}$ and $\vec{b} = 2\hat{\imath} - \hat{\jmath} + \lambda\hat{k}$, then the value of λ is:

(a) 2

(b) -2

(c) 4

(d) 1

16. The shortest distance between two parallel lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}$ is given by:

(a) $\frac{|\vec{b}\cdot(\vec{a}_2-\vec{a}_1)|}{|\vec{b}|}$

(b) $\frac{|(\vec{a}_2-\vec{a}_1)\times\vec{b}|}{|\vec{b}|}$

(c) $\frac{|\vec{a}_2-\vec{a}_1|\times|\vec{b}|}{|\vec{b}|}$

(d) $\frac{|\vec{b}\times(\vec{a}_2-\vec{a}_1)|}{|\vec{a}_2-\vec{a}_1|}$

17. The angle between the line $x/1 = y/0 = z/-1$ and the plane $x + y = 0$ is:

(a) 0°

(b) 45°

(c) 60°

(d) 90°

18. The corner points of the feasible region are $(0,0), (3,0), (1,3)$ and $(0,2)$. If the objective function is $Z = 4x + 6y$, the minimum value of Z is:

(a) 0

(b) 12

(c) 22

(d) 10

Assertion-Reasoning Based Questions

In questions 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer from the following options:

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. **Assertion (A):** If $P(A) = 0.6, P(B) = 0.5$ and $P(A \cap B) = 0.3$, then A and B are independent events. **Reason (R):** Two events A and B are independent if $P(A \cap B) = P(A)P(B)$.

20. **Assertion (A):** The derivative of e^{x^2} is $2xe^{x^2}$. **Reason (R):** The derivative of $e^{f(x)}$ is $e^{f(x)}f'(x)$ by the Chain Rule.

SECTION B (10 Marks)

This section comprises **5** *questions of* **2** *marks each.*

21. If $y = \sec^{-1}\left(\frac{1}{2x^2-1}\right)$, find $\frac{dy}{dx}$.

22. Find the value of p so that the lines $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are perpendicular.

OR

Find a vector of magnitude 5 units parallel to the resultant of the vectors $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$ and $\vec{b} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$.

23. Evaluate $\int \frac{\sin x}{1+\cos x}\, dx$.

OR

Find the slope of the tangent to the curve $x = a\cos^3\theta, y = a\sin^3\theta$ at $\theta = \frac{\pi}{4}$.

24. If $A = \begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$, verify that $(A+B)^T = A^T + B^T$.

25. A problem in mathematics is given to three students A, B, C whose chances of solving it are $1/2, 1/3, 1/4$ respectively. Find the probability that the problem is solved.

SECTION C (18 Marks)

This section comprises **6** *questions of* **3** *marks each.*

26. Show that the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \frac{2x-1}{3}$ is invertible. Also, find $f^{-1}(x)$.

27. Evaluate $\int \frac{x+2}{\sqrt{4x-x^2}}\, dx$.

OR

Evaluate $\int e^x \left(\frac{2+\sin 2x}{1+\cos 2x}\right) dx$.

28. Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + \sin\left(\frac{y}{x}\right)$, given that $y = \frac{\pi}{2}$ when $x = 1$.

OR

A wire of length 36 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a rectangle whose length is twice its breadth. What should be the lengths of the two pieces so that the combined area is minimum?

29. Find the distance of the point $P(-2, 4, -5)$ from the line $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$.

OR

Find the equation of the plane through the point $(1, 0, -2)$ and perpendicular to the planes $2x + y - z = 2$ and $x - y - z = 3$.

30. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, show that $A^2 - 4A + I = O$, where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

31. Find the maximum value of $Z = 5x + 10y$ subject to $x + 2y \leq 120, x + y \geq 60, x - 2y \geq 0$, and $x \geq 0, y \geq 0$.

SECTION D (20 Marks)

This section comprises **4** *questions of* **5** *marks each.*

32. Using integration, find the area of the region bounded by the parabola $y = x^2$ and the line $y = x$.

OR

Evaluate $\int_0^1 \log(1 + x)\, dx$.

33. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & -1 \end{bmatrix}$, find A^{-1}. Hence, solve the system of equations $x + y + z = 6, x - y + z = 2, 2x + y - z = 1$.

34. An open tank with a square base and vertical sides is to be constructed to hold a given quantity of water. Show that the cost of material will be least when the depth is half of the width.

OR

Evaluate $\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}}\, dx$.

35. Find the shortest distance between the lines whose vector equations are:

$$\vec{r} = (1 - t)\hat{i} + (t - 2)\hat{j} + (3 - 2t)\hat{k}$$

$$\vec{r} = (s + 1)\hat{i} + (2s - 1)\hat{j} - (2s + 1)\hat{k}$$

SECTION E (12 Marks)

This section comprises **3** *case study based questions of* **4** *marks each.*

36. **Case Study 1: Total Cost and Marginal Cost**

The total cost of production of x units of a product is given by $C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$. Marginal Cost ($MC$) is the rate of change of total cost with respect to output x.

Based on the given information, answer the following questions:

(a) Find the Marginal Cost (MC) function. (1 Mark)

(b) Calculate the Marginal Cost when 3 units are produced. (3 Marks)

OR

(c) Find the average cost function $AC(x)$ and the value of fixed cost. (3 Marks)

37. **Case Study 2: Binomial Distribution in Trials**

In a college, it is observed that 40% of the students smoke. A random sample of 5 students is selected from the college. Let X be the random variable representing the number of smokers in the sample.

Based on the given information, answer the following questions:

(a) State the number of Bernoulli trials and the probability of success. (1 Mark)

(b) Find the probability that exactly 3 students are smokers. (3 Marks)

OR

(c) Find the probability that none of the students are smokers. (3 Marks)

38. **Case Study 3: Communication Tower and Plane**

A cellular tower is modelled as a line passing through the point $P(2, -1, 3)$ and parallel to the vector $3\hat{i} + 2\hat{j} - 5\hat{k}$. The reception boundary is modelled by the plane $x + 2y - z = 4$.

Based on the given information, answer the following questions:

(a) Write the vector equation of the line representing the tower. (1 Mark)

(b) Find the distance of the point $P(2, -1, 3)$ from the reception boundary plane. (3 Marks)

OR

(c) Find the equation of the plane passing through $P(2, -1, 3)$ and perpendicular to the tower (line). (3 Marks)