PRACTICE QUESTION PAPER - XIII
CLASS XII - MATHEMATICS (041)

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper contains **38** questions. All questions are compulsory.
2. The question paper is divided into FIVE Sections – A, B, C, D and E.
3. Section **A** comprises of **20** questions of **1** mark each. (18 MCQs + 2 Assertion-Reasoning)
4. Section **B** comprises of **5** questions of **2** marks each.
5. Section **C** comprises of **6** questions of **3** marks each.
6. Section **D** comprises of **4** questions of **5** marks each.
7. Section **E** comprises of **3** Case Study Based Questions of **4** marks each.
8. There is no overall choice in the question paper. However, an internal choice has been provided in **2** questions in Section B, **3** questions in Section C, **2** questions in Section D and **2** questions in Section E (in the sub-parts).
9. Use of calculators is **not** permitted.

SECTION A (20 Marks)

This section comprises **20** *questions of* **1** *mark each. Questions 1-18 are Multiple Choice Questions (MCQs) and questions 19-20 are Assertion-Reason based questions.*

Multiple Choice Questions (MCQs) and Assertion-Reason Questions (Combined Enumeration)

1. Let $f : \mathbb{N} \to \mathbb{N}$ be defined by $f(x) = x^2 + x + 1$. The function f is:
   (a) One-one and onto
   (b) One-one but not onto
   (c) Onto but not one-one
   (d) Neither one-one nor onto
2. The number of bijective functions from a set A containing 5 elements to itself is:
   (a) 5
   (b) 25
   (c) 120
   (d) 3125
3. The value of $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right)$ is:
   (a) $\pi/4$
   (b) $\pi/2$
   (c) $\tan^{-1}\left(\frac{5}{6}\right)$
   (d) $\tan^{-1}\left(\frac{1}{6}\right)$
4. If $\tan^{-1} x + \tan^{-1} y = \frac{\pi}{4}$ and $xy < 1$, then $x + y + xy$ is equal to:
   (a) 0
   (b) 1
   (c) -1
   (d) 2

5. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, then $A^2 - 4A$ is equal to:

   (a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

   (b) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

   (c) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

   (d) $\begin{bmatrix} 4 & 6 \\ 2 & 4 \end{bmatrix}$

6. If A is a square matrix, $A - A^T$ is a:

   (a) Symmetric matrix

   (b) Skew-symmetric matrix

   (c) Hermitian matrix

   (d) Identity matrix

7. If A is a non-singular matrix of order 3, and $|adj(A)| = 49$, then $|A|$ is:

   (a) 7

   (b) ± 7

   (c) 49

   (d) 1/7

8. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then the value of $|2A|$ is:

   (a) $2|A|$

   (b) $4|A|$

   (c) $8|A|$

   (d) $|A|$

9. The derivative of $\log(7x)$ with respect to x is:

   (a) $7/x$

   (b) $1/(7x)$

   (c) $1/x$

   (d) 7

10. A particle moves along the curve $y = \frac{2}{3}x^3 + 1$. The points on the curve where the y-coordinate is changing twice as fast as the x-coordinate are:

   (a) $(1, 5/3)$

   (b) $(\pm 1, 5/3)$

   (c) $(0, 1)$

   (d) $(1, 5/3)$ and $(-1, 1/3)$

11. $\int \frac{\sin x}{\sin(x-a)}\, dx$ is equal to:

   (a) $\sin a \log|\sin(x-a)| + x \cos a + C$

   (b) $x \cos a - \sin a \log|\sin(x-a)| + C$

   (c) $x \cos a + \sin a \log|\sin(x-a)| + C$

(d) $x \sin a + \cos a \log|\sin(x-a)| + C$

12. $\int_{-1}^{1} \frac{dx}{x^2+1}$ is equal to:

(a) $\pi/2$

(b) $\pi/4$

(c) π

(d) 0

13. The integrating factor of the differential equation $x\frac{dy}{dx} - y = \log x$ is:

(a) $1/x$

(b) x

(c) e^x

(d) e^{-x}

14. The scalar projection of $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\hat{i} + 2\hat{j} + \hat{k}$ is:

(a) $10/\sqrt{6}$

(b) $10/6$

(c) $\sqrt{6}/10$

(d) $6/10$

15. If $|\vec{a}| = 1$, then $|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2$ is equal to:

(a) 0

(b) 1

(c) 2

(d) 3

16. The direction cosines of the y-axis are:

(a) $(0, 1, 0)$

(b) $(1, 0, 0)$

(c) $(0, 0, 1)$

(d) $(1, 1, 1)$

17. The angle between the planes $2x - y + z = 6$ and $x + y + 2z = 7$ is:

(a) 0

(b) $\pi/6$

(c) $\pi/3$

(d) $\pi/2$

18. The region represented by the constraints $x \geq 0, y \geq 0, 2x + 3y \leq 6$ and $x + 4y \leq 4$ is:

(a) Bounded

(b) Unbounded

(c) Feasible

(d) Infeasible

Assertion-Reasoning Based Questions

In questions 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer from the following options:

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. **Assertion (A):** If A and B are independent events, then $P(A \cap B | A \cup B) = \frac{P(A)P(B)}{1-P(A')P(B')}$. **Reason (R):** For independent events A and B, $P(A \cap B) = P(A)P(B)$.

20. **Assertion (A):** The maximum value of $\frac{\log x}{x}$ is $\frac{1}{e}$. **Reason (R):** $\frac{d}{dx}\left(\frac{\log x}{x}\right)$ is zero at $x = e$.

SECTION B (10 Marks)

This section comprises **5** *questions of* **2** *marks each.*

21. Find $\frac{dy}{dx}$ if $y = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$, $-\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$.

22. If the volume of the parallelepiped whose coterminous edges are $2\hat{\imath}-3\hat{\jmath}+4\hat{k}$, $\hat{\imath}+2\hat{\jmath}-\hat{k}$ and $3\hat{\imath}-\hat{\jmath}+\lambda\hat{k}$ is 5 cubic units, find the value of λ.

OR

Find the direction cosines of the line passing through the points $(-2, 4, -5)$ and $(1, 2, 3)$.

23. Evaluate $\int \frac{1}{\sqrt{4x-x^2}}\, dx$.

OR

Find the approximate change in the volume of a cube of side x meters caused by increasing the side by 1%.

24. If $\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 6 \\ -2 & 4 \end{vmatrix}$, write the value of x.

25. If A and B are two events such that $P(A) = 0.5, P(B) = 0.6$ and $P(A \cap B) = 0.3$. Find $P(A' \cap B')$.

SECTION C (18 Marks)

This section comprises **6** *questions of* **3** *marks each.*

26. Show that $\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x-y}{x+y}\right) = \frac{\pi}{4}$.

27. Evaluate $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx$.

OR

Evaluate $\int \frac{x^2}{x^4+x^2+1}\, dx$.

28. Find the general solution of the differential equation $\cos x \frac{dy}{dx} + y \sin x = 1$.

OR

Find the equation of the tangent and normal to the curve $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at the point $(0, 5)$.

29. Find the vector equation of the line passing through the point $(1, 2, -4)$ and perpendicular to the two lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.

OR

Find the foot of the perpendicular from the point $(1, 2, 3)$ to the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$.

30. Using properties of determinants, prove that $\begin{vmatrix} 1 & x & y \\ 1 & x+y & y \\ 1 & x & y+x \end{vmatrix} = x^2y$.

31. Maximize $Z = 5x + 7y$ subject to $2x + y \leq 8, x + 2y \leq 10, x \geq 0, y \geq 0$.

SECTION D (20 Marks)

This section comprises **4** *questions of* **5** *marks each.*

33. Using integration, find the area of the region bounded by the circle $x^2 + y^2 = a^2$.

OR

Evaluate $\int \frac{1}{x^4+1}\,dx$.

34. Given $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, find A^{-1}. Hence, solve the system of equations $x-y+z = 4$, $2x+y-3z = 0$, $x + y + z = 2$.

35. Find the maximum volume of the cylinder inscribed in a sphere of radius R.

OR

Evaluate $\int \frac{\sin x}{\sin 4x}\,dx$.

36. Find the shortest distance between the lines:

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$$

$$\vec{r} = (2\hat{i} + 4\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 4\hat{j} + 5\hat{k})$$

SECTION E (12 Marks)

This section comprises **3** *case study based questions of* **4** *marks each.*

37. **Case Study 1: Rate of Change in Geometry**

A man of height 2 m walks at a uniform speed of 5 km/h away from a lamppost which is 6 m high. Let x be the distance of the man from the lamppost and L be the length of his shadow.

Based on the given information, answer the following questions:

(a) By using similar triangles, write the relation between x and L. (1 Mark)

(b) Find the rate at which the length of his shadow is increasing. (3 Marks)

OR

(c) Find the rate at which the tip of the shadow is moving. (3 Marks)

38. **Case Study 2: Bayes' Theorem in Medical Testing**

A person is known to have suffered from a certain disease. The probability that a patient, chosen at random from the population, has the disease is 0.1%. The diagnostic test for this disease is 99% accurate (i.e., if a person has the disease, the test is positive 99% of the time, and if a person does not have the disease, the test is negative 99% of the time).

Based on the given information, answer the following questions:

(a) What is the probability that a person is healthy (H) and the test result is positive (P)? (1 Mark)

(b) Find the probability that a randomly chosen person gets a positive test result. (3 Marks)

OR

(c) If a person gets a positive test result, find the probability that they actually have the disease (D). (3 Marks)

39. **Case Study 3: Diet Problem Formulation**

A dietician wishes to mix two types of food, X and Y, in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 10 units of vitamin B. Food X costs Rs 2 per unit and Food Y costs Rs 3 per unit. Food X contains 2 units of vitamin A and 1 unit of vitamin B per unit. Food Y contains 1 unit of vitamin A and 2 units of vitamin B per unit. The dietician wants to minimize the cost of the mixture.

Based on the given information, answer the following questions:

(a) Define the objective function Z to be minimized. (1 Mark)

(b) Write the constraint inequalities based on the minimum vitamin requirements. (3 Marks)

OR

(c) State the non-negativity constraints for the variables. (3 Marks)
