Pair of Linear Equations in Two Variables

Mathematics Study Material

PART 1: Introduction and Basic Concepts

Theory (150 words)

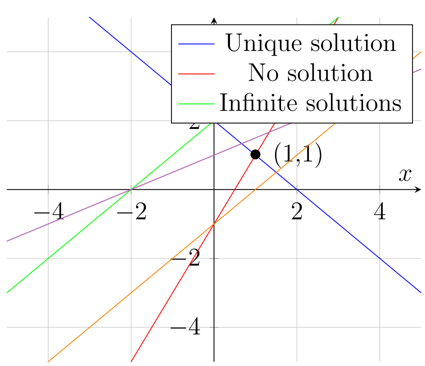
A pair of linear equations in two variables represents two straight lines on the Cartesian plane. The general form is:

$$\begin{cases} a_1x + b_1y + c_1 = 0 \\ a_2x + b_2y + c_2 = 0 \end{cases}$$

where $a_1, b_1, c_1, a_2, b_2, c_2$ are real numbers, and x and y are variables.

The solution to this system is the ordered pair (x, y) that satisfies both equations simultaneously. Geometrically, this represents the point of intersection of the two lines. There are three possibilities:

1. **Unique solution**: The lines intersect at one point (consistent system)
2. **No solution**: The lines are parallel and distinct (inconsistent system)
3. **Infinitely many solutions**: The lines coincide (dependent system)



Solved Problems

1. Determine whether the following pair of equations has a unique solution, no solution, or infinitely many solutions:

$$2x + 3y = 7 \quad \text{and} \quad 4x + 6y = 14$$

Solution:
Rewrite the equations in standard form:

$$2x + 3y - 7 = 0 \quad \text{and} \quad 4x + 6y - 14 = 0$$

Compare the ratios of coefficients:

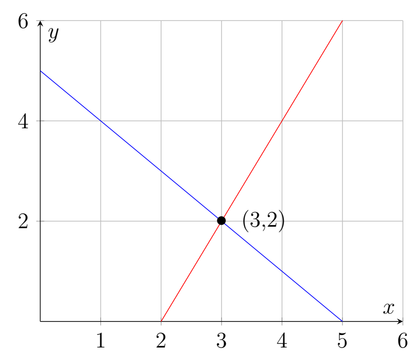
$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}, \quad \frac{c_1}{c_2} = \frac{-7}{-14} = \frac{1}{2}$$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, the equations represent coincident lines with infinitely many solutions.

2. Solve graphically:

$$x + y = 5 \quad \text{and} \quad 2x - y = 4$$

Solution:



The lines intersect at (3,2), which is the solution.

3. Solve by substitution method:

$$3x - y = 3 \quad \text{and} \quad 2x + 3y = 11$$

Solution:
From first equation: $y = 3x - 3$
Substitute into second equation:

$$2x + 3(3x - 3) = 11 \Rightarrow 2x + 9x - 9 = 11 \Rightarrow 11x = 20 \Rightarrow x = \frac{20}{11}$$

Then $y = 3(\frac{20}{11}) - 3 = \frac{60}{11} - \frac{33}{11} = \frac{27}{11}$
Solution: $\left(\frac{20}{11}, \frac{27}{11}\right)$

4. Solve by elimination method:

$$5x + 2y = 16 \quad \text{and} \quad 3x - 4y = 2$$

Solution:
Multiply first equation by 2:

$$10x + 4y = 32$$

Add to second equation:

$$(10x + 4y) + (3x - 4y) = 32 + 2 \Rightarrow 13x = 34 \Rightarrow x = \frac{34}{13}$$

Substitute back:

$$5(\frac{34}{13}) + 2y = 16 \Rightarrow \frac{170}{13} + 2y = \frac{208}{13} \Rightarrow 2y = \frac{38}{13} \Rightarrow y = \frac{19}{13}$$

Solution: $\left(\frac{34}{13}, \frac{19}{13}\right)$

5. Solve by cross-multiplication:

$$2x + 3y = 7 \quad \text{and} \quad 3x + 2y = 8$$

Solution:
Using cross-multiplication formula:

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Here:

$$\frac{x}{(3)(8) - (2)(7)} = \frac{y}{(7)(3) - (8)(2)} = \frac{1}{(2)(2) - (3)(3)}$$

$$\frac{x}{24 - 14} = \frac{y}{21 - 16} = \frac{1}{4 - 9}$$

$$\frac{x}{10} = \frac{y}{5} = \frac{1}{-5}$$

Thus:

$$x = \frac{10}{-5} = -2, \quad y = \frac{5}{-5} = -1$$

Solution: $(-2, -1)$

6. The sum of two numbers is 35 and their difference is 5. Find the numbers.

Solution:
Let the numbers be x and y.
Then:

$$x + y = 35 \quad \text{and} \quad x - y = 5$$

Adding both equations:

$$2x = 40 \Rightarrow x = 20$$

Then $y = 35 - 20 = 15$
The numbers are 20 and 15.

Self Practice Problems (15 problems)

1. Solve graphically: $2x + y = 6$ and $2x - y = 2$ [Answer: (2,2)]
2. Solve by substitution: $x + 2y = 5$ and $3x - y = 1$ [Answer: (1,2)]
3. Solve by elimination: $3x + 4y = 10$ and $2x - 2y = 2$ [Answer: (2,1)]
4. Solve by cross-multiplication: $5x + 3y = 11$ and $2x + 4y = 10$ [Answer: (1,2)]
5. Find whether the pair has unique solution: $3x + 2y = 5$ and $6x + 4y = 10$ [Answer: Infinitely many solutions]
6. The cost of 5 pens and 3 pencils is Rs. 35, while 3 pens and 2 pencils cost Rs. 22. Find cost of each. [Answer: Pen=Rs.4, Pencil=Rs.5]
7. A fraction becomes $\frac{1}{3}$ when 1 is subtracted from both numerator and denominator, and becomes $\frac{1}{2}$ when 1 is added to both. Find the fraction. [Answer: $\frac{3}{7}$]
8. Solve: $\frac{x}{2} + \frac{y}{3} = 2$ and $\frac{x}{3} + \frac{y}{2} = \frac{13}{6}$ [Answer: (1,3)]
9. Find the value of k for which the system $2x + ky = 1$ and $3x - 5y = 7$ has a unique solution. [Answer: $k \neq -\frac{10}{3}$]
10. The sum of digits of a two-digit number is 9. If 27 is added, the digits reverse. Find the number. [Answer: 36]
11. Solve: $0.2x + 0.3y = 1.3$ and $0.4x + 0.5y = 2.3$ [Answer: (2,3)]
12. Find the value of p for which the system $3x + py = 0$ and $6x + 4y = 0$ has non-trivial solutions. [Answer: 2]
13. A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it goes 40 km upstream and 55 km downstream. Find the speed of boat in still water and the speed of the stream. [Answer: Boat=8 km/h, Stream=3 km/h]
14. Solve: $\frac{2}{x} + \frac{3}{y} = 13$ and $\frac{5}{x} - \frac{4}{y} = -2$ [Answer: $x = \frac{1}{2}, y = \frac{1}{3}$]
15. Find the values of a and b for which the system $(a-1)x+3y = 2$ and $6x+(1-2b)y = 6$ has infinitely many solutions. [Answer: $a = 3, b = -4$]

PART 2: Graphical Solution

Theory (120 words)

The graphical method of solving a pair of linear equations involves plotting both equations on the same coordinate plane and finding their point of intersection.

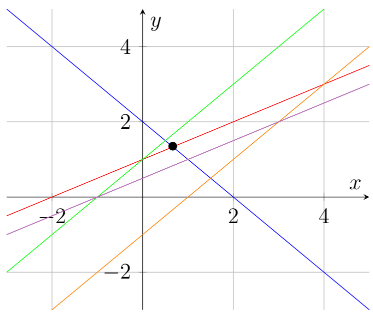
Steps for graphical solution:

1. Find at least two solutions for each equation

2. Plot these points on graph paper
3. Draw straight lines through the points
4. The intersection point gives the solution

Possible outcomes:

- **Intersecting lines**: One unique solution (consistent system)
- **Parallel lines**: No solution (inconsistent system)
- **Coincident lines**: Infinitely many solutions (dependent system)



Solved Problems

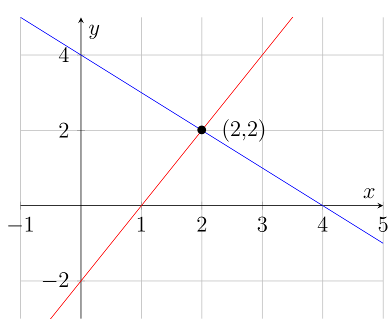
1. Solve graphically: $x + y = 4$ and $2x - y = 2$

   Solution:
   For $x + y = 4$:

x	y
0	4
4	0

   For $2x - y = 2$:

x	y
0	-2
1	0



The lines intersect at (2,2), which is the solution.

2. Determine graphically whether the pair has a solution:

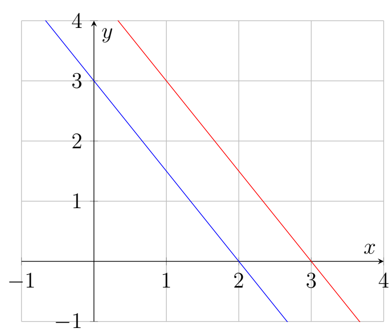
$$3x + 2y = 6 \quad \text{and} \quad 6x + 4y = 18$$

Solution:
For $3x + 2y = 6$:

x	y
0	3
2	0

For $6x + 4y = 18$:

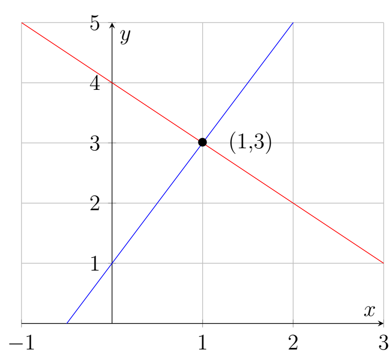
x	y
1	3
3	0



The lines are parallel and distinct, so there is no solution.

3. Solve graphically: $y = 2x + 1$ and $y = -x + 4$

Solution:



The lines intersect at (1,3), which is the solution.

4. Show graphically that the system has infinitely many solutions:

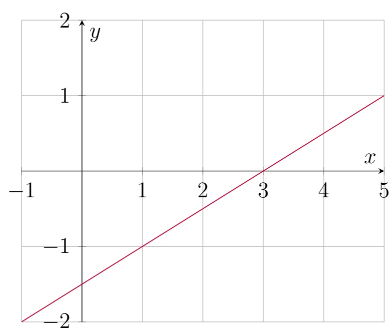
$$x - 2y = 3 \quad \text{and} \quad 3x - 6y = 9$$

Solution:
For $x - 2y = 3$:

x	y
3	0
1	-1

For $3x - 6y = 9$ (which simplifies to $x - 2y = 3$):

x	y
3	0
1	-1



Both equations represent the same line, so there are infinitely many solutions.

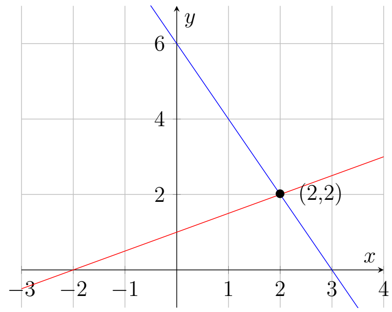
5. Solve graphically: $2x + y = 6$ and $x - 2y = -2$

Solution:
For $2x + y = 6$:

x	y
0	6
3	0

For $x - 2y = -2$:

x	y
0	1
-2	0



The lines intersect at (2,2), which is the solution.

6. Determine graphically whether the system is consistent:

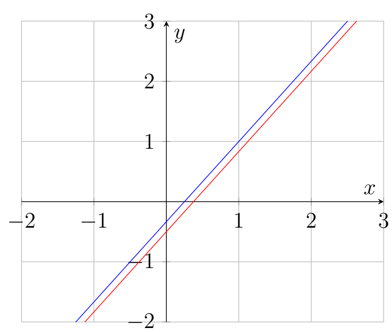
$$4x - 3y = 1 \quad \text{and} \quad 8x - 6y = 3$$

Solution:
For $4x - 3y = 1$:

x	y
1	1
-0.5	-1

For $8x - 6y = 3$:

x	y
0	-0.5
1.5	1.5



The lines are parallel and distinct, so the system is inconsistent (no solution).

Self Practice Problems (15 problems)

1. Solve graphically: $x + y = 5$ and $2x - y = 1$ [Answer: (2,3)]

2. Solve graphically: $3x + 2y = 12$ and $x - y = -1$ [Answer: (2,3)]

3. Determine graphically whether the system has a solution: $2x+3y=5$ and $4x+6y=15$ [Answer: No solution]
4. Solve graphically: $y=3x-2$ and $y=-x+6$ [Answer: (2,4)]
5. Show graphically that $x-y=1$ and $2x-2y=2$ have infinitely many solutions. [Answer: Coincident lines]
6. Solve graphically: $5x+2y=16$ and $3x+4y=18$ [Answer: (2,3)]
7. Determine graphically whether $x+2y=4$ and $2x+4y=12$ have a solution. [Answer: No solution]
8. Solve graphically: $\frac{x}{2}+\frac{y}{3}=1$ and $\frac{x}{3}+\frac{y}{2}=1$ [Answer: (6/5,6/5)]
9. Solve graphically: $0.5x+0.25y=1$ and $2x+y=4$ [Answer: Infinitely many solutions]
10. Solve graphically: $3x-4y=7$ and $5x+2y=3$ [Answer: (1,-1)]
11. Determine graphically whether $6x-3y=9$ and $2x-y=3$ have a unique solution. [Answer: Infinitely many solutions]
12. Solve graphically: $x+3y=6$ and $2x-y=5$ [Answer: (3,1)]
13. Solve graphically: $y=4x-5$ and $y=2x+1$ [Answer: (3,7)]
14. Determine graphically whether $4x+5y=20$ and $8x+10y=40$ have a unique solution. [Answer: Infinitely many solutions]
15. Solve graphically: $2x+5y=13$ and $3x-2y=4$ [Answer: (2,1.8)]

PART 3: Algebraic Methods

Theory (150 words)

There are three main algebraic methods to solve a pair of linear equations:

1. **Substitution Method**:

- Solve one equation for one variable
- Substitute this expression into the other equation
- Solve for the remaining variable
- Back-substitute to find the other variable

2. **Elimination Method**:

- Multiply equations by suitable numbers to make coefficients of one variable equal
- Add or subtract equations to eliminate one variable

- Solve for the remaining variable
- Substitute back to find the other variable

3. **Cross-Multiplication Method**:

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

This formula directly gives the solution by computing the cross-products of coefficients.

When to use which method?
- Substitution: When one variable has coefficient 1
- Elimination: When coefficients are simple
- Cross-multiplication: For complex coefficients

Solved Problems

1. Solve by substitution:

$$2x + y = 7 \quad \text{and} \quad 4x - 3y = -1$$

Solution:
From first equation: $y = 7 - 2x$
Substitute into second equation:

$$4x - 3(7 - 2x) = -1 \Rightarrow 4x - 21 + 6x = -1 \Rightarrow 10x = 20 \Rightarrow x = 2$$

Then $y = 7 - 2(2) = 3$
Solution: (2,3)

2. Solve by elimination:

$$3x + 4y = 10 \quad \text{and} \quad 2x - 2y = 2$$

Solution:
Multiply second equation by 2:

$$4x - 4y = 4$$

Add to first equation:

$$(3x + 4y) + (4x - 4y) = 10 + 4 \Rightarrow 7x = 14 \Rightarrow x = 2$$

Substitute back:

$$3(2) + 4y = 10 \Rightarrow 6 + 4y = 4 \Rightarrow y = 1$$

Solution: (2,1)

3. Solve by cross-multiplication:

$$3x - 2y = 5 \quad \text{and} \quad 4x + 3y = 11$$

Solution:
Using the formula:

$$\frac{x}{(-2)(11) - (3)(5)} = \frac{y}{(5)(4) - (11)(3)} = \frac{1}{(3)(3) - (4)(-2)}$$

$$\frac{x}{-22 - 15} = \frac{y}{20 - 33} = \frac{1}{9 + 8}$$

$$\frac{x}{-37} = \frac{y}{-13} = \frac{1}{17}$$

Thus:

$$x = \frac{-37}{17}, \quad y = \frac{-13}{17}$$

Solution: $\left(\frac{-37}{17}, \frac{-13}{17}\right)$

4. Solve:

$$\frac{x}{2} + \frac{y}{3} = 1 \quad \text{and} \quad \frac{x}{3} + \frac{y}{2} = \frac{11}{6}$$

Solution:
Let $\frac{1}{x} = u$ and $\frac{1}{y} = v$:
$3u + 2v = 1$ and $2u + 3v = \frac{11}{6}$
Multiply first by 3 and second by 2:

$$9u + 6v = 3 \quad \text{and} \quad 4u + 6v = \frac{11}{3}$$

Subtract:

$$5u = 3 - \frac{11}{3} = -\frac{2}{3} \Rightarrow u = -\frac{2}{15}$$

Then $3(-\frac{2}{15}) + 2v = 1 \Rightarrow -\frac{6}{15} + 2v = 1 \Rightarrow 2v = \frac{21}{15} \Rightarrow v = \frac{7}{10}$
Thus $x = -\frac{15}{2}$ and $y = \frac{10}{7}$
Solution: $\left(-\frac{15}{2}, \frac{10}{7}\right)$

5. Solve:

$$0.2x + 0.3y = 1.3 \quad \text{and} \quad 0.4x + 0.5y = 2.3$$

Solution:
Multiply both equations by 10:

$$2x + 3y = 13 \quad \text{and} \quad 4x + 5y = 23$$

Multiply first equation by 2:

$$4x + 6y = 26$$

Subtract second equation:

$$(4x + 6y) - (4x + 5y) = 26 - 23 \Rightarrow y = 3$$

Then $2x + 3(3) = 13 \Rightarrow 2x = 4 \Rightarrow x = 2$
Solution: (2,3)

6. Solve:

$$\frac{x+y}{xy} = 2 \quad \text{and} \quad \frac{x-y}{xy} = 6$$

Solution:
Rewrite as:

$$\frac{1}{y} + \frac{1}{x} = 2 \quad \text{and} \quad \frac{1}{y} - \frac{1}{x} = 6$$

Let $u = \frac{1}{x}$ and $v = \frac{1}{y}$:
$v + u = 2$ and $v - u = 6$
Add: $2v = 8 \Rightarrow v = 4$
Subtract: $2u = -4 \Rightarrow u = -2$
Thus $x = -\frac{1}{2}$ and $y = \frac{1}{4}$
Solution: $\left(-\frac{1}{2}, \frac{1}{4}\right)$

Self Practice Problems (15 problems)

1. Solve by substitution: $3x - y = 3$ and $9x - 3y = 9$ [Answer: Infinitely many solutions]
2. Solve by elimination: $5x + 3y = 11$ and $4x - y = 2$ [Answer: (1,2)]
3. Solve by cross-multiplication: $2x + 3y = 7$ and $3x + 2y = 8$ [Answer: (2,1)]
4. Solve: $\frac{2}{x} + \frac{3}{y} = 5$ and $\frac{5}{x} - \frac{4}{y} = -3$ [Answer: $x = 1, y = 1$]
5. Solve: $0.3x + 0.5y = 0.5$ and $0.5x + 0.7y = 0.74$ [Answer: (0.8,0.52)]
6. Solve: $\frac{x+y}{xy} = 1$ and $\frac{x-y}{xy} = 3$ [Answer: $x = \frac{1}{2}, y = -1$]
7. Solve: $3(x + y) = 7xy$ and $3(x - y) = -xy$ [Answer: $x = 1, y = \frac{3}{4}$]
8. Solve: $\frac{x}{2} + \frac{y}{3} = 2$ and $\frac{x}{3} + \frac{y}{2} = \frac{13}{6}$ [Answer: (1,3)]
9. Solve: $\frac{x+y-8}{2} = \frac{x+2y-14}{3} = \frac{3x+y-12}{11}$ [Answer: (2,6)]
10. Solve: $6x + 5y = 7x + 3y + 1 = 2(x + 6y - 1)$ [Answer: (1,1)]
11. Find k for which the system $2x + ky = 1$ and $3x - 5y = 7$ has no solution. [Answer: $k = -\frac{10}{3}$]
12. Find p for which the system $3x + py = 0$ and $6x + 4y = 0$ has non-trivial solutions. [Answer: 2]

13. Solve: $2x + 3y = 17$ and $2x + 2 - 3y + 1 = 5$ [Answer: (4,3)]

14. Solve: $\frac{x+1}{2} + \frac{y-1}{3} = 8$ and $\frac{x-1}{3} + \frac{y+1}{2} = 9$ [Answer: (10,14)]

15. Solve: $\frac{5}{x+y} + \frac{2}{x-y} = 3$ and $\frac{15}{x+y} - \frac{7}{x-y} = 10$ [Answer: $x = 3, y = 2$]

PART 4: Applications in Daily Life

Theory (150 words)

Linear equations in two variables have numerous real-world applications:

1. **Finance**: Calculating profit/loss, interest rates, investment planning 2. **Business**: Determining break-even points, cost analysis 3. **Physics**: Solving problems related to motion, forces, etc. 4. **Geometry**: Finding dimensions of geometric shapes 5. **Mixtures**: Determining quantities in mixture problems 6. **Time-Distance**: Calculating speeds and distances

Key steps for solving application problems:

1. Identify the variables and assign them
2. Translate the given conditions into equations
3. Solve the system of equations
4. Interpret the solution in the original context

Common problem types
- Age problems
- Number problems
- Money/coin problems
- Distance-speed-time problems
- Work-rate problems
- Mixture problems

Solved Problems

1. The sum of two numbers is 35 and their difference is 5. Find the numbers.

   Solution:
   Let the numbers be x and y.
   Then:
   $$x + y = 35 \quad \text{and} \quad x - y = 5$$
   Adding: $2x = 40 \Rightarrow x = 20$
   Then $y = 35 - 20 = 15$
   The numbers are 20 and 15.

2. A fraction becomes $\frac{1}{2}$ when 1 is subtracted from both numerator and denominator, and becomes $\frac{2}{3}$ when 1 is added to both. Find the fraction.

   Solution:
   Let the fraction be $\frac{x}{y}$.
   Then:
   $$\frac{x-1}{y-1} = \frac{1}{2} \quad \text{and} \quad \frac{x+1}{y+1} = \frac{2}{3}$$
   Cross-multiplying:
   $$2x - 2 = y - 1 \quad \text{and} \quad 3x + 3 = 2y + 2$$
   Simplifying:
   $$2x - y = 1 \quad \text{and} \quad 3x - 2y = -1$$
   Solving gives $x = 3$ and $y = 5$
   The fraction is $\frac{3}{5}$.

3. A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it goes 40 km upstream and 55 km downstream. Find the speed of the boat in still water and the speed of the stream.

   Solution:
   Let boat speed = x km/h, stream speed = y km/h
   Then:
   $$\frac{30}{x-y} + \frac{44}{x+y} = 10 \quad \text{and} \quad \frac{40}{x-y} + \frac{55}{x+y} = 13$$
   Let $u = \frac{1}{x-y}$ and $v = \frac{1}{x+y}$:
   $30u + 44v = 10$ and $40u + 55v = 13$
   Solving gives $u = \frac{1}{5}$ and $v = \frac{1}{11}$
   Thus $x - y = 5$ and $x + y = 11$
   Therefore $x = 8$ km/h and $y = 3$ km/h.

4. The sum of digits of a two-digit number is 9. If 27 is added, the digits reverse. Find the number.

   Solution:
   Let the number be $10x + y$ where x is tens digit and y is units digit.
   Then:
   $$x + y = 9 \quad \text{and} \quad 10x + y + 27 = 10y + x$$
   Simplifying second equation:
   $$9x - 9y = -27 \Rightarrow x - y = -3$$
   Solving with first equation:
   $$x + y = 9 \quad \text{and} \quad x - y = -3$$
   Adding: $2x = 6 \Rightarrow x = 3$
   Then $y = 6$
   The number is 36.

5. The cost of 5 pens and 3 pencils is Rs. 35, while 3 pens and 2 pencils cost Rs. 22. Find the cost of each.

   Solution:
   Let cost of pen $= x$ Rs, pencil $= y$ Rs
   Then:
   $$5x + 3y = 35 \quad \text{and} \quad 3x + 2y = 22$$
   Multiply first by 2 and second by 3:
   $$10x + 6y = 70 \quad \text{and} \quad 9x + 6y = 66$$
   Subtract: $x = 4$
   Then $5(4) + 3y = 35 \Rightarrow 3y = 15 \Rightarrow y = 5$
   Cost: pen = Rs.4, pencil = Rs.5

6. A man has Rs. 200 in denominations of Rs. 5 and Rs. 10 notes. If the number of Rs. 10 notes is twice that of Rs. 5 notes, find how many notes of each he has.

   Solution:
   Let number of Rs.5 notes $= x$, Rs.10 notes $= y$
   Then:
   $$5x + 10y = 200 \quad \text{and} \quad y = 2x$$
   Substitute second into first:
   $$5x + 10(2x) = 200 \Rightarrow 25x = 200 \Rightarrow x = 8$$
   Then $y = 16$
   He has 8 Rs.5 notes and 16 Rs.10 notes.

Self Practice Problems (15 problems)

1. The sum of two numbers is 45 and their difference is 15. Find the numbers. [Answer: 30,15]

2. A fraction becomes $\frac{2}{3}$ when 1 is added to both numerator and denominator, and becomes $\frac{1}{2}$ when 1 is subtracted from both. Find the fraction. [Answer: $\frac{3}{5}$]

3. A boat goes 24 km upstream and 28 km downstream in 6 hours. It goes 30 km upstream and 21 km downstream in 6.5 hours. Find the speed of boat in still water and the stream. [Answer: Boat=10 km/h, Stream=2 km/h]

4. The sum of digits of a two-digit number is 8. If 18 is added, the digits reverse. Find the number. [Answer: 35]

5. The cost of 3 apples and 4 bananas is Rs. 32, while 5 apples and 2 bananas cost Rs. 34. Find the cost of each. [Answer: Apple=Rs.6, Banana=Rs.3.5]

6. A man has Rs. 300 in denominations of Rs. 10 and Rs. 20 notes. If the number of Rs. 20 notes is one more than twice the Rs. 10 notes, find how many notes of each he has. [Answer: Rs.10=8, Rs.20=17]

7. Five years ago, a man was seven times as old as his son. Five years hence, he will be three times as old. Find their current ages. [Answer: Man=40, Son=10]

8. A train covers a certain distance at uniform speed. If speed were 5 km/h more, it would take 1 hour less, and if 5 km/h less, it would take 1 hour more. Find distance and speed. [Answer: Distance=300 km, Speed=25 km/h]

9. A chemist has two solutions of acid, one 40

10. The perimeter of a rectangle is 44 cm and its area is 120 cm². Find its dimensions. [Answer: 12cm × 10cm]

11. A man can row 15 km downstream and 10 km upstream in 5 hours. He can row 30 km downstream and 20 km upstream in 10 hours. Find his rowing speed and the stream speed. [Answer: Rowing=5 km/h, Stream=2 km/h]

12. The sum of a number and its reciprocal is $\frac{10}{3}$. Find the number. [Answer: 3 or $\frac{1}{3}$]

13. Points A and B are 90 km apart. Two cars start from A and B towards each other and meet after 1 hour. If they move in same direction, they meet in 9 hours. Find their speeds. [Answer: 50 km/h, 40 km/h]

14. A two-digit number is 4 times the sum of its digits. If 18 is added, the digits reverse. Find the number. [Answer: 24]

15. A person invested Rs. 10,000 in two schemes at 8

PART 5: Self Assessment Test (25 MCQs)

1. The pair of equations $x + 2y = 5$ and $3x + 6y = 15$ has:

   (a) A unique solution
   (b) No solution
   (c) Infinitely many solutions
   (d) Exactly two solutions

   Answer: (c)

2. The value of k for which the system $3x + 5y = 0$ and $kx + 10y = 0$ has non-trivial solutions is:

   (a) 6
   (b) 0

(c) 2
(d) 5

Answer: (a)

3. The solution of $\frac{x}{2} + \frac{y}{3} = 1$ and $\frac{x}{3} + \frac{y}{2} = \frac{11}{6}$ is:

   (a) (2,3)
   (b) (3,2)
   (c) (1,3)
   (d) (3,1)

   Answer: (c)

4. The pair of equations $y = 0$ and $y = -7$ has:

   (a) One solution
   (b) Two solutions
   (c) No solution
   (d) Infinitely many solutions

   Answer: (c)

5. The value of k for which the system $x + 2y = 3$ and $5x + ky = 15$ has infinitely many solutions is:

   (a) 5
   (b) 10
   (c) 6
   (d) 15

   Answer: (b)

6. The sum of digits of a two-digit number is 9. If 27 is added, the digits reverse. The number is:

   (a) 36
   (b) 63
   (c) 45
   (d) 54

   Answer: (a)

7. The solution of $0.2x + 0.3y = 1.3$ and $0.4x + 0.5y = 2.3$ is:

(a) (2,3)
(b) (3,2)
(c) (1,3)
(d) (3,1)

Answer: (a)

8. The cost of 2 pencils and 3 erasers is Rs. 9, while 4 pencils and 6 erasers cost Rs. 18. This represents:

   (a) Unique solution
   (b) No solution
   (c) Infinitely many solutions
   (d) None of these

   Answer: (c)

9. The pair of equations $x = a$ and $y = b$ graphically represents lines that are:

   (a) Parallel
   (b) Intersecting at (a,b)
   (c) Coincident
   (d) None of these

   Answer: (b)

10. The solution of $2x + 3y = 7$ and $6x + 9y = 21$ is:

   (a) Unique
   (b) No solution
   (c) Infinite
   (d) None

   Answer: (c)

11. The value of k for which the system $x + ky = 1$ and $kx + y = 1$ has no solution is:

   (a) 1
   (b) -1
   (c) 0
   (d) No such k exists

   Answer: (d)

12. The solution of $\frac{x+y}{xy} = 2$ and $\frac{x-y}{xy} = 6$ is:

   (a) $\left(\frac{1}{4}, -\frac{1}{2}\right)$
   (b) $\left(-\frac{1}{2}, \frac{1}{4}\right)$
   (c) $\left(\frac{1}{2}, -\frac{1}{4}\right)$
   (d) $\left(-\frac{1}{4}, \frac{1}{2}\right)$

   Answer: (b)

13. The pair of equations $3x + 2y = 5$ and $2x - 3y = 7$ has:

   (a) One solution
   (b) Two solutions
   (c) No solution
   (d) Infinitely many solutions

   Answer: (a)

14. The value of k for which the lines $5x + 2y = k$ and $10x + 4y = 3$ are parallel is:

   (a) $\frac{3}{2}$
   (b) $\frac{2}{3}$
   (c) $\frac{3}{4}$
   (d) $\frac{4}{3}$

   Answer: (a)

15. The solution of $2x + 5y = 13$ and $3x - 2y = 4$ is:

   (a) (2,1.8)
   (b) (1.8,2)
   (c) (2,1.6)
   (d) (1.6,2)

   Answer: (a)

16. The pair of equations $x + 3y = 6$ and $2x - y = 5$ has solution:

   (a) (3,1)
   (b) (1,3)
   (c) (2,2)
   (d) (4,1)

   Answer: (a)

17. The value of k for which the system $2x + 3y = 5$ and $4x + ky = 10$ has infinitely many solutions is:

    (a) 2
    (b) 4
    (c) 6
    (d) 8

    Answer: (c)

18. The solution of $\frac{2}{x} + \frac{3}{y} = 13$ and $\frac{5}{x} - \frac{4}{y} = -2$ is:

    (a) $x = 1, y = \frac{1}{3}$
    (b) $x = \frac{1}{2}, y = \frac{1}{3}$
    (c) $x = 1, y = \frac{1}{2}$
    (d) $x = \frac{1}{3}, y = 1$

    Answer: (b)

19. The pair of equations $y = 4x - 5$ and $y = 2x + 1$ has solution:

    (a) (3,7)
    (b) (7,3)
    (c) (2,5)
    (d) (5,2)

    Answer: (a)

20. The value of p for which the system $3x + py = 0$ and $6x + 4y = 0$ has non-trivial solutions is:

    (a) 1
    (b) 2
    (c) 3
    (d) 4

    Answer: (b)

21. The solution of $3x - 4y = 7$ and $5x + 2y = 3$ is:

    (a) (1,-1)
    (b) (-1,1)
    (c) (1,1)
    (d) (-1,-1)

Answer: (a)

22. The pair of equations $4x + 5y = 20$ and $8x + 10y = 40$ has:

    (a) Unique solution
    (b) No solution
    (c) Infinitely many solutions
    (d) None

    Answer: (c)

23. The solution of $\frac{x+1}{2} + \frac{y-1}{3} = 8$ and $\frac{x-1}{3} + \frac{y+1}{2} = 9$ is:

    (a) (10,14)
    (b) (14,10)
    (c) (12,12)
    (d) (8,16)

    Answer: (a)

24. The value of k for which the system $x + 2y = 3$ and $5x + ky = -7$ has no solution is:

    (a) 5
    (b) 10
    (c) 15
    (d) 20

    Answer: (b)

25. The solution of $\frac{5}{x+y} + \frac{2}{x-y} = 3$ and $\frac{15}{x+y} - \frac{7}{x-y} = 10$ is:

    (a) $x = 3, y = 2$
    (b) $x = 2, y = 3$
    (c) $x = 4, y = 1$
    (d) $x = 1, y = 4$

    Answer: (a)

PART 6: Self Assessment Paper

Section A: MCQs (8 questions)

1. The pair of equations $x + y = 4$ and $2x + 2y = 8$ has:
   (a) Unique solution
   (b) No solution
   (c) Infinitely many solutions
   (d) None

   Answer: (c)

2. The value of k for which the system $2x + 3y = 5$ and $4x + ky = 10$ has infinitely many solutions is:
   (a) 2
   (b) 4
   (c) 6
   (d) 8

   Answer: (c)

3. The solution of $0.5x + 0.25y = 1$ and $2x + y = 4$ is:
   (a) (1,2)
   (b) (2,1)
   (c) No solution
   (d) Infinitely many solutions

   Answer: (d)

4. The pair of equations $y = 3x + 2$ and $y = 3x - 1$ has:
   (a) One solution
   (b) No solution
   (c) Infinitely many solutions
   (d) None

   Answer: (b)

5. The solution of $\frac{x}{3} + \frac{y}{4} = 1$ and $\frac{x}{2} - \frac{y}{3} = -1$ is:
   (a) (0,4)
   (b) (4,0)

(c) (-6,12)
(d) (12,-6)

Answer: (c)

6. The value of p for which the system $3x + py = 0$ and $6x + 4y = 0$ has non-trivial solutions is:

   (a) 1
   (b) 2
   (c) 3
   (d) 4

   Answer: (b)

7. The solution of $3x - 4y = 7$ and $5x + 2y = 3$ is:

   (a) (1,-1)
   (b) (-1,1)
   (c) (1,1)
   (d) (-1,-1)

   Answer: (a)

8. The pair of equations $4x + 5y = 20$ and $8x + 10y = 40$ has:

   (a) Unique solution
   (b) No solution
   (c) Infinitely many solutions
   (d) None

   Answer: (c)

Section B: Short Answer Questions (6 questions)

1. Solve by substitution: $3x - y = 3$ and $9x - 3y = 9$ [Answer: Infinitely many solutions]
2. Solve by elimination: $5x + 3y = 11$ and $4x - y = 2$ [Answer: (1,2)]
3. Solve by cross-multiplication: $2x + 3y = 7$ and $3x + 2y = 8$ [Answer: (2,1)]
4. Find k for which the system $2x + ky = 1$ and $3x - 5y = 7$ has no solution. [Answer: $k = -\frac{10}{3}$]
5. The sum of digits of a two-digit number is 9. If 27 is added, the digits reverse. Find the number. [Answer: 36]
6. Solve: $0.3x + 0.5y = 0.5$ and $0.5x + 0.7y = 0.74$ [Answer: (0.8,0.52)]

Section C: Long Answer Questions (4 questions)

1. Solve graphically: $2x + y = 6$ and $2x - y = 2$ and verify algebraically. [Answer: (2,2)]
2. A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it goes 40 km upstream and 55 km downstream. Find the speed of boat in still water and the speed of the stream. [Answer: Boat=8 km/h, Stream=3 km/h]
3. Solve: $\frac{5}{x+y} + \frac{2}{x-y} = 3$ and $\frac{15}{x+y} - \frac{7}{x-y} = 10$ [Answer: $x = 3, y = 2$]
4. Five years ago, a man was seven times as old as his son. Five years hence, he will be three times as old. Find their current ages. [Answer: Man=40, Son=10]

Section D: Case Study (120 words)

A chemistry lab has two types of acid solutions. Solution A contains 30% acid and solution B contains 60% acid. The lab technician wants to prepare 100 liters of a 45% acid solution by mixing solutions A and B.

Let x liters be the amount of solution A and y liters be the amount of solution B needed. The total volume equation is $x + y = 100$. The acid content equation is $0.3x + 0.6y = 0.45 \times 100$.

Based on this information, answer the following questions:

1. The system of equations representing this situation is:
   (a) $x + y = 100$ and $3x + 6y = 45$
   (b) $x + y = 100$ and $0.3x + 0.6y = 45$
   (c) $x + y = 100$ and $30x + 60y = 45$
   (d) $x + y = 100$ and $3x + 6y = 450$

   Answer: (b)
2. The simplified form of the acid content equation is:
   (a) $x + 2y = 150$
   (b) $x + 2y = 100$
   (c) $2x + y = 150$
   (d) $2x + y = 100$

   Answer: (a)
3. The solution to the system is:
   (a) $x = 50, y = 50$
   (b) $x = 40, y = 60$
   (c) $x = 60, y = 40$

(d) $x = 30, y = 70$

Answer: (a)

4. If the technician wants a 50% solution instead, the new acid equation would be:

   (a) $0.3x + 0.6y = 50$

   (b) $0.3x + 0.6y = 100$

   (c) $0.3x + 0.6y = 500$

   (d) $0.3x + 0.6y = 0.5$

   Answer: (a)

5. For the 50% solution, the required quantities would be:

   (a) $x = \frac{100}{3}, y = \frac{200}{3}$

   (b) $x = 50, y = 50$

   (c) $x = 25, y = 75$

   (d) $x = 75, y = 25$

   Answer: (a)

PART 7: Quick Revision Table

Pair of Linear Equations in Two Variables	
Concept	**Key Points**
General Form	$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$
Solution Types	Unique (intersecting), None (parallel), Infinite (coincident)
Condition for Unique Solution	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
Condition for No Solution	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
Condition for Infinite Solutions	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
Substitution Method	Solve one equation for one variable, substitute into other
Elimination Method	Make coefficients equal, add/subtract to eliminate variable
Cross-Multiplication	$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$
Graphical Solution	Plot both lines, intersection point is solution
Applications	Age, number, money, distance-speed-time, mixture problems